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ABSTRACT

This concept module on force transformers is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. This module describes specific force transformers in the mechanical translational, mechanical rotational, fluidal, and electrical systems and discusses their fundamental similarity as transformers of forcelike quantities. Included with the concept module are eight application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Levers, Wedges and Inclined Planes, Drive Systems, Gear Trains, Block and Tackle, Differential Hoist, Hydraulic Press, and Electrical Transformers. (LRA)

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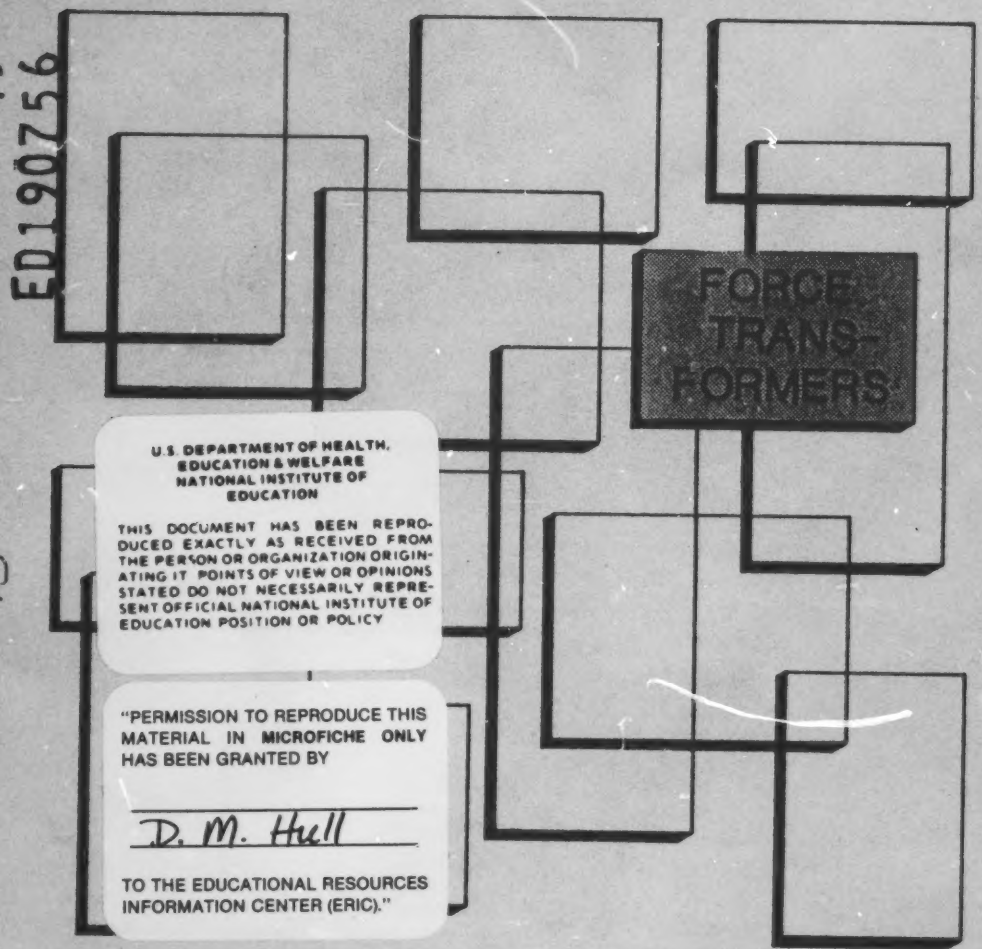
UNIFIED TECHNICAL CONCEPTS

CONCEPT MODULE 8-Q

ED190756



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INTRODUCTION

There are times when nature seems to give something for nothing. However, closer examination when comparing some output to some input shows that nature insists on the principles of conservation of energy. More energy cannot be taken out of a system than put into the system.

This module treats a class of devices called "force transformers" because, in such devices, the force output is different from the force input. However, the work, or energy output, is never greater than the work, or energy, input; and with an ideal case in which no energy is lost to frictional forces, the work output equals the work input.

Force transformers are simply machines having great diversity and having a wide variety of applications, but the basic explanation of their usefulness is the same. The unified approach taken in the preceding modules will assist in understanding force transformers in different energy systems as examples of the same physical principle.

PREREQUISITES

Student should have completed Concept Module 6-0, "Power."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe specific force transformers in the mechanical translational, mechanical rotational, fluidal, and electrical systems; and discuss their fundamental similarity as transformers of forcelike quantities.
- Define the following terms:

  Ideal mechanical advantage.
  Actual mechanical advantage.
  Efficiency.

- Calculate the ideal mechanical advantage of a specific pulley, lever, screw, wheel and axle, hydraulic press or lift, and electrical transformer.
- Calculate the change in current in an ideal electrical transformer.
- Discuss how the role of resistance in a transformer dissipates energy input and reduces efficiency.
- Describe the power input and power output characteristics of a transformer that operates continuously.

INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems <u>force</u> and <u>torque</u> cause motion of mass.
- In fluidal systems <u>pressure difference</u> causes motion of fluids.
- In electrical systems <u>potential difference</u> causes motion of mass.
- In thermal systems <u>temperature difference</u> causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

COMPONENTS OF UTC

UTC incorporates three types of modules.

CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

INTRODUCTION

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

PREREQUISITES

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

OBJECTIVES

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

DISCUSSION

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

LABORATORY (called DEMONSTRATIONS in the Concept Modules)

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

SELF CHECK

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

REFERENCES

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS

8-0 Force Transformers

8M1 Levers

8M2 Wedges and Inclined Planes

8M3 Drive Systems

8M4 Gear Trains

8M5 Block and Tackle

8M6 Differential Hoist

8F1 Hydraulic Press

8E1 Electrical Transformers

PRINCIPLES OF FORCE TRANSFORMERS

The prospect of changing a flat tire without a lever jack would be a dismal one. With a jack, however, the relatively weak muscular forces of the human body are transformed, or magnified, sufficiently to raise one end of a two-ton car off the ground. Levers and screw jacks are examples of force transformers in mechanical systems.

An example of force transformers in fluid systems is a barber's chair. When the barber wishes to elevate the customer to a more convenient height, the barber makes use of a hydraulic pump. With a few effortless motions of a lever, the client is lifted in the chair.

Consider, finally, force transformers in electrical systems. Good examples of these devices are electric calculators and cassette recorders that operate from a 110-volt line. Ordinarily, these devices operate from batteries, which typically provide 6 volts. The accomodation is made by "stepping down" the 110 volts to the required 6 volts by use of a "6-volt transformer." Here, voltage, rather than mechanical force, is being transformed; but recall that in the electrical system, voltage is the "forcelike quantity." Thus, the lever/screw jack and the 6-volt transformer appear to be similar physical systems.

In each of the examples considered above there is a coupling device, or transformer, that receives an input from some source and produces an output fed to some load, as shown schematically in Figure 1.

FIG. 1

Re-examination of the three examples of force transformers given above according to this scheme shows that in the first example, the source is an individual operating the jack. The individual's input is the muscular force exerted by their arm. The coupling device is the screw jack. The load is the car, and the output is the force required to lift one end of the car off the ground. (These facts are summarized in Figure 2.)

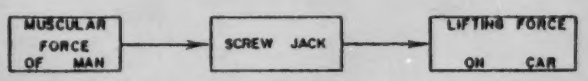


FIG. 2

In this case, the function of the screw jack is to multiply the input force sufficiently to lift the car with the required output force.

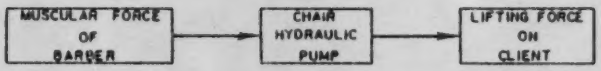


FIG. 3

The other examples can be diagrammed in the same way (Figures 3 and 4).

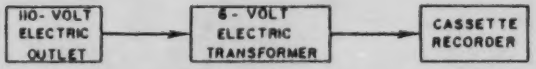


FIG. 4

Energy is produced by work done at the source, and is consumed as work done on the load. Recall the following from Module 2-0:

WORK = FORCELIKE QUANTITY X DISPLACEMENTLIKE QUANTITY (1)

The coupling device, or force transformer, transfers the energy from the source to the load. Although the forcelike quantities and the displacementlike quantities at each end may have different magnitudes, the product of these quantities (which is the energy transferred from input to output) is ideally the same. In practice, the output energy will be somewhat less, because some of the energy will be dissipated as heat from friction in the operation of the transformer itself. Energy is conserved overall, but some is wasted in overcoming resistance. Naturally, one tries to reduce these losses in order to make a highly-efficient transformer. The percent efficiency of a transformer is defined by Equation 2.

$$\eta = \frac{E_{out}}{E_{in}} (100) \qquad (2)$$

where

η = % efficiency.
E_{out} = useful output energy.
E_{in} = input energy.
100 = multiplier to obtain %.

When useful output energy equals input energy (the ideal case), no energy is lost in the coupling device; and the formula gives 100% efficiency. In actual cases, the output energy is somewhat less, and the formula gives an efficiency of less than 100%. In no case can the efficiency be greater than 100%.

In the discussion that follows, ideal, or 100%, efficiency will be assumed, unless otherwise specified. Keep in mind that real transformers will be less efficient, because of energy losses in the actual operation. The actual efficiency cannot be predicted in advance, because it will depend on how successively the device can be made to operate without frictional affects.

MECHANICAL FORCE TRANSFORMERS

THE PULLEY

A simple pulley is shown in Figure 5. A weight W is lifted by pulling on the cord with a force F. Experimentation with this system shows that the required force F is equal to the weight W if there is no friction in the pulley; otherwise, F must be greater than W. This arrangement does not multiply force; it is not, therefore, a force transformer. About all that can be said of this device is that it changes the direction of the force. When the rope is pulled downward, the weight is lifted upward. The device may be useful, but it is not a force transformer. The pulley pictured in Figure 6 is, however, a force transformer.

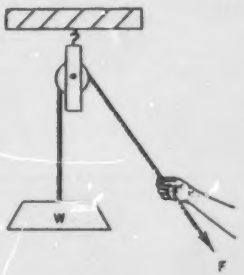


FIG. 5

Study the figure, and notice that if the rope is pulled (or F_2 moves) a distance of 1 foot, the load will be raised (or F_1 moves) only 1/2 foot. Thus, x_2 will always be twice x_1, or $x_1/x_2 = 1/2$. When the arrangement is used for an ideal transformer, input work = output work, as stated by Equation 3.

$$F_2x_2 = F_1x_1$$

or

$$F_2 = (x_1/x_2)\, F_1 \qquad (3)$$

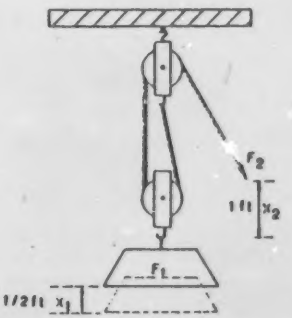


FIG. 6

where

- F_2 = input force at the source.
- F_1 = output force at the load = weight to be lifted.
- x_2 = vertical distance moved by input force.
- x_1 = vertical distance moved by output force or weight.

The necessary conclusion is that $F_2 = 1/2\ F_1$ and that the weight is lifted by pulling with a force of only 1/2 the weight. This pulley arrangement definitely is a force transformer. One "pays" for the force advantage by being required to pull twice as far as the weight actually rises. Notice that this arrangement makes the product of force and displacement the same at the source and load ends. Study this argument carefully, because it is the

basic argument that applies to all ideal force transformers: What one gains as a force advantage, one gives up as a displacement disadvantage; but the product of force and displacement, representing work, must be the same at input and output.

Because a load twice as large as the applied force can be lifted with the pulley in Figure 6, this pulley "has a mechanical advantage of 2." In general, for an ideal transformer, mechanical advantage is given by Equation 4. Equation 4 is used to solve problems in Examples A and C.

$$M_i = \frac{x_2}{x_1} \tag{4}$$

where

M_i = ideal mechanical advantage.

From Equation 3, notice that the ratio of x_2/x_1 is identical with the ratio F_1/F_2 for an ideal transformer. Clearly, then, the ratio F_1/F_2 could be used as well as x_2/x_1 to determine mechanical advantage. In an actual pulley, since there will be some friction involved, the input force will have to be slightly larger to do the job. The mechanical advantage F_1/F_2 in practice then would have to be determined by measurements of both F_2 and F_1 in an actual test of the pulley. For an actual transformer, the mechanical advantage is given by Equation 5, used to solve problems in Examples A, B, and C.

$$M_a = \frac{F_1}{F_2} \tag{5}$$

where

M_a = actual mechanical advantage.

If the transformer is frictionless, either formula will give the same result for mechanical advantage.

A little algebra will show that the formula for % efficiency (Equation 2) is equivalent to various other convenient forms. Any of the following forms given in Equation 6 can be used as well:

$$\eta = \frac{M_a}{M_i}(100) \tag{6}$$

or

$$\eta = \frac{F_2(\text{ideal})}{F_2(\text{actual})}(100)$$

or

$$\eta = \frac{F_1\,(\text{actual})}{F_1\,(\text{ideal})}\ (100)$$

EXAMPLE A: THE PULLEY AS A FORCE TRANSFORMER

Given: In the pulley system shown in Figure 6, a load of 400 lb is lifted 5 ft. (Assume that this system is an ideal transformer.)

Find: (a) The amount of work that is done by the source, (b) the applied force F_2, and (c) distance that the input force moves.

Solution: In this case, the pulley has an ideal mechanical advantage of 2.

(a) Output work: $W_{out} = F_1 x_1$

$= (444)(5)$

$W_{out} = 2{,}000$ ft·lb

(b) Applied force: $F_2 = \frac{F_1}{M}$

$= \frac{400}{2}$

$F_2 = 200$ lb

(c) Since $M_I = x_2/x_1$, the input force displacement corresponding to the load displacement of 5 ft can be found from:

$x_2 = (M_I)\ x_1 = (2)(5\text{ ft}) = 10$ ft

EXAMPLE B: MECHANICAL ADVANTAGE AND EFFICIENCY OF PULLEY

Given: When actually measured, the applied force in Example A is found to be 225 lb.

Find: (a) The actual M of the pulley and (b) its % efficiency.

Solution: (a) $M_a = \frac{F_1}{F_2}$

$= \frac{400}{225}$

$M_a = 1.78$

(b) $\eta = \frac{M_a}{M_i}(100)$ (from Equation 6)

$= (1.78/2)(100)$

$\eta = 89\%$

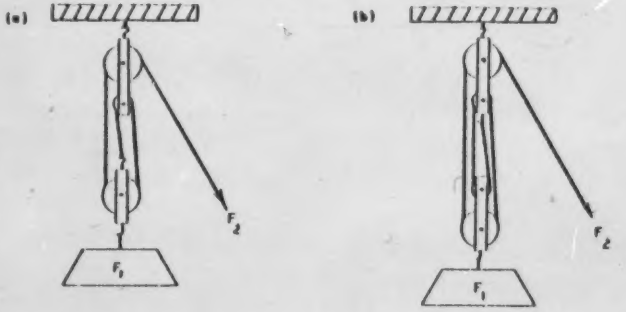
EXAMPLE C: MECHANICAL ADVANTAGE OF BLOCK AND TACKLE

Given: Pulley systems a and b as shown. A combination of pulleys of this type is called a "block and tackle."

Find: The ideal mechanical advantage of the pulleys shown.

Solution: (a) F_2 must move down vertically three times as much as F_1 moves up, if the cord is not to stretch. Thus, $M_i = x_2/x_1 = 3/1 = 3$

(b) In this case, by similar reasoning, $M_i = x_2/x_1 = 4/1 = 4$. Notice that the ideal M_i turns out to be just the number of lengths or cord supporting the load, not counting the one being pulled on directly.



THE LEVER

The lever, essentially, is a rigid bar that can be pivoted at some point. Consider the crowbar (Figure 7a) used to move a heavy rock. A smaller stone serves as the pivot point. The input force F_2 is transformed to an output force F_1 large enough to move the boulder. Figure 7b illustrates the lever with dimensions before and after the boulder is raised. The dotted line indicates the final position of the lever. Notice that the applied force F_2 moves through a larger distance x_2 than the load, which is lifted a distance x_1. The force advantage is gained, as with the pulley, at the expense of displacement.

FIG. 7

The mechanical advantage of the ideal lever can be found from the formula arrived at in Equation 4. When we refer to Figure 7, we conclude that $M_I = x_2/x_1$. This equation can be put into a more useful form by relating the displacements to the lever arms of the lever; that is, the distance from each end to the pivot point. Notice that angles θ_2 and θ_1 are equal, so that —

$$\frac{x_2}{x_1} = \frac{\ell_2}{\ell_1}.$$

Since the two ratios are the same, one can write Equation 7 as follows:

$$M_I = \frac{\ell_2}{\ell_1} \qquad (7)$$

where

ℓ_1 = lever arm of load force.
ℓ_2 = lever arm of applied force.

According to Equation 7, ℓ_2 must be greater than ℓ_1 to magnify the force input. The greater force advantage achieved, the larger the ratio of ℓ_2 to ℓ_1, or the nearer the load to the pivot point. (See Examples D and E, where Equation 7 is used to solve problems.

EXAMPLE D: MECHANICAL ADVANTAGE OF A LEVER

Given: The boulder of Figure 7 weighs 500 lb, and the input force on the 6'0" crowbar cannot be greater than 100 lb.

Find: The point at which the pivot stone should be placed to move the boulder.

Solution: The lever must transform an input force of 100 lb to an output force of 500 lb; therefore, the required mechanical advantage is 500/100 = 5. Then —

$$M_i = 5 = \frac{\ell_2}{\ell_1}$$

or

$$\ell_2 = 5\,\ell_1$$

Since $\ell_2 + \ell_1 = 6$ ft (the length of the crowbar), the algebraic solution of the last two equations much be $\ell_2 = 5$ ft, and $\ell_1 = 1$ ft. Thus, the pivot must be no further than 1 ft from the load end of the lever.

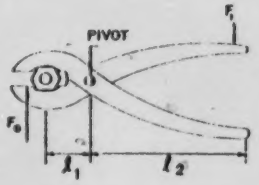
EXAMPLE E: PLIERS AS A LEVER

Given: A pair of pliers.

Find: The reason why a pair of pliers is effective in removing a stubborn nut.

Solution: The pliers actually is a two-bar lever. If one of the handles (say, the lower one) is considered fixed, then the mechanical advantage is as follows:

$$M_i = \frac{\ell_2}{\ell_1} > 1$$



THE INCLINED PLANE

A crate is moved more easily into the back end of a truck by use of an incline than by a direct vertical lift (Figure 8). The inclined plane serves as another coupling device that enables a small force to do a big job at the expense of a force displacement. In this case, the crate is rolled along the incline further than it is lifted; that is, $x_2 > x_1$. This "sacrifice" in displacement shows up as a gain in force, or a mechanical advantage. Work done by the source and in lifting the load is indicated thusly:

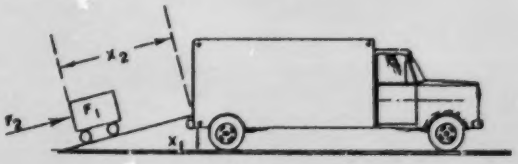


FIG. 8

$$W_2 = F_2 x_2$$

$$W_1 = F_1 x_1$$

where

W_2 = work done by source.
W_1 = work done in lifting the load.

If the rollers are well-greased in such a manner that the process is practically friction free, the input work is nearly equal to the output work, or —

$$F_2 x_2 = F_1 x_1.$$

Then the load force $F_1 = (x_2/x_1)\, F_2$. The mechanical advantage giving a force magnification of F_1/F_2 can be written for an inclined plane in the form of Equation 8.

$$M_I = \frac{x_2}{x_1} \qquad (8)$$

where

x_1 = rise of incline.
x_2 = length of incline.

Equation 8 is employed in Example F to solve a problem.

Notice that this equation is just another form of Equation 4. The ratio of incline to rise sometimes is called the "grade" of the incline, so that the M_I of an incline plane is easily remembered as simply the grade of the incline. (See Examples F and G.) This ideal M_I would not be achieved if it were necessary to slide (rather than to roll) the crate up the incline. Why?

EXAMPLE F: MECHANICAL ADVANTAGE OF AN INCLINED PLANE

Given: A 300-lb safe is pushed up an incline 15 ft long and 5 ft high. (Assume that the rollers are frictionless.)

Find: (a) The grade of the incline, (b) the M_i of the incline, and (c) the force F_2 required to roll the safe up the incline.

Solution:

(a) $M_i = \frac{x_2}{x_1}$

$= \frac{15}{5}$

$M_i = 3$

(b) M_i = grade

$M_i = 3$

(c) $F_2 = \frac{F_1}{F_2}$

$= \frac{15}{5}$

$F_2 = 3$

or

$F_2 = \frac{F_1}{e}$

$= \frac{300}{3}$

$F_2 = 100$ lb

EXAMPLE G: THE WEDGE AS A AN INCLINED PLANE

Given: A "wedge" used to split logs.

Find: Why the wedge makes the job easier.

Solution: The wedge is one variation, or application, of the inclined plane. Notice that the wedge is just two inclined planes joined together back to back. Its M_i allows the use of a driving force which actually is smaller than the sideways force required to split the log.

THE SCREW

The screw is a clever adaptation of the inclined plane in the rotational mechanical system. The ordinary wood screw (Figure 9), for example, really is an inclined plane wrapped into a spiral around the screw axis.

FIG. 9

In order to appreciate the transforming advantage of the screw, consider a type screw jack used to raise the corner of a house (Figure 10). Rotation of the lever forces the loading platform on the screw upwards, lifting the load. Notice that this device combines two force transformers — the lever and the screw. When the handle is turned by an input force F_2 through the distance of one revolution, the load rises by a much smaller distance, equal to the linear displacement by one thread of the screw. This distance between threads of the screw usually is called the screw "pitch." If the screw has 5 threads per linear inch, for example, the load will be lifted 1/5 inch for each revolution of the screw.

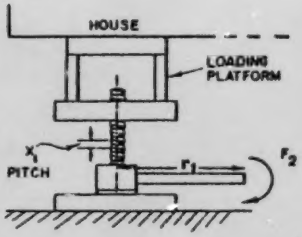


FIG. 10

Equation 4 is used again for ideal M_I. Now the relative displacement of the force F_2 and of the load F_1 must be determined. For one revolution of the handle, the force moves through one circumference of a circle with radius r_2, as shown. The corresponding displacement of the load is equal to the pitch of the screw. Therefore, Equation 9 can be written thusly for the ideal M_I of a screw.

$$M_I = \frac{2\pi r_2}{x_1} \qquad (9)$$

where

$2\pi r_2$ = circumference of input force of circle.
π = pi, 3.14.
r_1 = radius of circle formed by rotating handle.
x_1 = pitch of screw (distance between threads).

In such applications of the screw, the load usually is so large that frictional forces of turning are appreciable, even when the screw is well-lubricated. As a result, the actual M_I is considerably less than that predicted by Equation 9. As explained previously, the actual M_a can only be determined by measuring the source force F_2 needed to move the load F_1. Then M_a(actual) = F_1/F_2(measured). Equation 9 is used to solve problems in Examples H and I.

EXAMPLE H: BENCH VISE AS A SCREW

Given: The screw in a bench vise has a pitch of 0.25 inch. A worker applies a tangential force of 30 lb on the handle at a point 8 inches from the axis of the screw.

Find: (a) The M_i of the vise, (b) the force that could be exerted by the vise jaws (under ideal conditions), and (c) the % efficiency of the vise. (When the force exerted by the vise jaws is actually measured, it is found to be only 4000 lb.)

Solution:

(a) $M_i = \frac{2\pi r_2}{x_1}$

$= 2(3.14)(8\text{ in})/0.25\text{ in}$

$M_i = 201$

(b) $F_1 = M_i F_2$

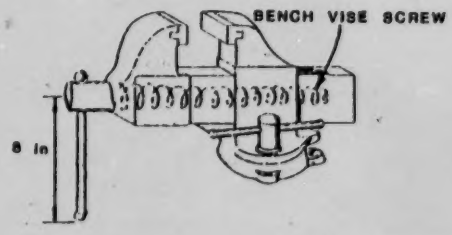
$= (201)(30\text{ lb})$

$F_1 = 6030\text{ lb}$

(c) $\eta = \frac{F_1(\text{actual})}{F_1(\text{ideal})}(100)$

$= (\frac{4000\text{ lb}}{6030\text{ lb}})(100)$

$\eta = 66.3\%$



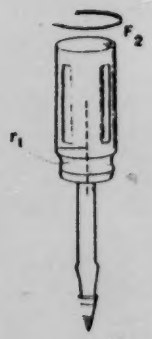


FIG. 11

In the ordinary use of a screwdriver, the radius r_2 is the radius of the driver handle, as shown in Figure 11.

EXAMPLE I: SCREW JACKS USED IN LIFTING A BUILDING

Given: Four screw jacks are placed under the corners of a square structure weighing 25 tons. The screw jacks are identical, each with a pitch of 0.20 in, and a handle 2.5 ft long, measured from the screw axis.

Find: (a) The M_I of each screw jack, (b) the force that would have to be applied to each screw jack to lift the load when all four are working together, assuming 100% efficiency, and (c) the force if the screw jacks are only 30% efficient.

Solution:

(a) $M_I = \frac{2\pi r_2}{x_1}$

$= 2(3.14)(30 \text{ in})/0.20 \text{ in}$

$M_I = 942$

(b) Each screw jack must take 1/4 the total load, or 25/4 tons = 6.25 tons. This value is (6.25)(2000) = 12,500 lb each. Then —

$F_2 = \frac{F_1}{M_I}$

$= 12,500/942$

$F_2 = 13.3 \text{ lb}$

(c) $\eta = \frac{F_2(\text{ideal})}{F_2(\text{actual})}$

$= \frac{F_2(\text{ideal})}{\eta}$

$= 13.3 \text{ lb}/0.30$

$\eta = 44.3 \text{ lb}$

THE WHEEL AND THE AXLE

When a wheel and axle are connected rigidly in such a manner that they turn together, another type of force transformer results in the rotational mechanical system. For example, Figure 12 shows two wheels of different radii, around which ropes are wound. One rope holds the load; the other is pulled by the input force F_2. To determine the M_I of this simple device, consider one revolution of the axle. The rope applying the input force moves a distance equal to the circumference of the large wheel, or $2\pi r_2$. The rope holding the weight or load moves through a distance equal to the circumference of the smaller wheel, or $2\pi r_1$.

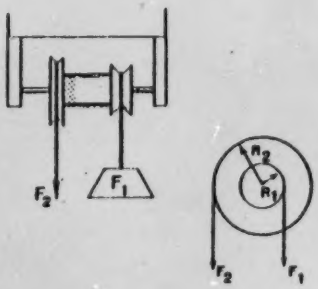


FIG. 12

When the principles of Equation 4 are applied, Equation 10 is derived to resolve this wheel-and-axle problem. An application is shown in Example J.

$$M_I = \frac{x_2}{x_1} = \frac{2\pi r_2}{2\pi r_1} = \frac{r_2}{r_1} \qquad (10)$$

where

r_1 = radius of load wheel.
r_2 = radius of source, or drive, wheel.

EXAMPLE J: MECHANICAL ADVANTAGE OF A WHEEL AND AXLE

Given: A bucket holding 30 lb of water is raised from a well by the type of wheel and axle shown. The turning handle has a length of 1.5 ft, and the rope holding the bucket is wrapped around a shaft of 6 inch diameter.

Find: The M_I and the required input force at the handle.

Solution:

$$M_I = \frac{r_2}{r_1}$$

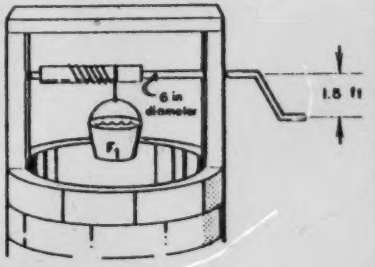
$$= \frac{18}{3}$$

$$M_I = 6$$

$$F_2 = \frac{F_1}{M_I}$$

$$= \frac{30}{6}$$

$$F_2 = 5 \text{ lb}$$



FLUIDAL FORCE TRANSFORMERS - THE HYDRAULIC LIFT

An example of a force transformer in the fluid system is the useful hydraulic lift. A diagram of a hydraulic jack is given in Figure 13, which illustrates a source of energy coupled to a load through an incompressible fluid-like oil. A heavy load, such as a 4000-lb automobile, can be lifted at the load piston by a much smaller force applied at the source piston. Again, this feat is possible because the applied force F_2 must move through a larger distance than the load. Suppose the source piston is pushed down a distance x_2. How far does the load piston rise? Since the fluid is practically incompressible, the volume of fluid V_2 displaced on the source side must show up on the load side V_1. This relationship can be written —

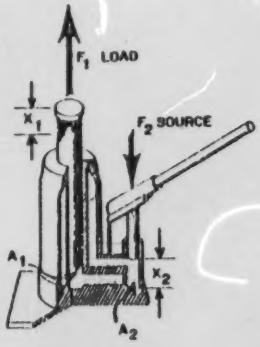


FIG. 13

$$V_2 = V_1$$

or

$$x_2A_2 = x_1A_1$$

where

V_1 = volume of fluid displaced on the load side.
V_2 = volume of fluid displaced on the source side.
A_1 = cross-sectional area of piston on the load side.
A_2 = cross-sectional area of piston on the source side.

The last equation leads to this proportion:

$$\frac{x_2}{x_1} = \frac{A_1}{A_2}$$

The mechanical advantage can be derived as an expression of the work in a fluid system as the product of the pressure and volume displacement; however, use of Equation 4 is simpler and more straightforward:

$$M_I = \frac{x_2}{x_1}$$

As shown in the previous equation, the ratio of displacement x_2/x_1 is the same as the ratio of areas, A_1/A_2. The M_I then can be written as Equation 11 for a hydraulic lift.

$$M_I = \frac{A_1}{A_2} \qquad (11)$$

Since little friction is involved in the operation of the hydraulic lift, the actual M_I can be expected to approach the ideal value given by this equation. An application is shown in Example K.

EXAMPLE K: MECHANICAL ADVANTAGE OF A HYDRAULIC LIFT

Given: A hydraulic lift contains a load piston 9 inches in diameter and a pump piston 1 inch in diameter.

Find: (a) The M_I of the lift under ideal conditions and (b) the force on the pump piston required to lift a total load of 5000 lb.

Solution:

(a) $$M_I = \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2}$$

$$= (4.5 \text{ in})^2/(0.5 \text{ in})^2$$

$$M_I = 81$$

(b) $$F_2 = \frac{F_1}{M_I}$$

$$= 5000 \text{ lb}/81$$

$$F_2 = 61.7 \text{ lb}$$

ELECTRICAL TRANSFORMERS

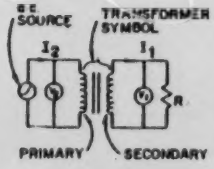


FIG. 14

In the electrical system, the forcelike quantity is voltage. The electrical transformer is a device that can step up or can step down voltage at the expense of current. The source in this case is producing alternating current, and the load is some device that consumes electrical energy. In Figure 14, the load is shown as a simple resistor R. (Notice the symbol for the transformer used in drawing electrical circuits.) V_2 and V_1 represent input and output voltages, respectively. Voltmeters placed in the positions shown would record the appropriate voltages. The transformer itself usually is a rectangular iron core with two windings — the primary windings on the source side and the secondary windings on the load side. Figure 15 consists of a simple drawing of a transformer in the same circuit.

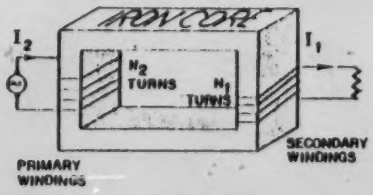


FIG. 15

As with the belt drive, source energy is fed to the electrical transformer at a continuous rate. Thus, in an ideal transformer, in which no energy losses occur, the following equality is true for power input = power output:

$$V_2 I_2 = V_1 I_1$$

where

V_1 = voltage output.
V_2 = voltage input.
I_1 = current output.
I_2 = current input.

(Refer to Concept Module 6-0 for the expression of power in the electrical system.)

The complete theory of the transformer is complicated, since it involves the magnetic fields produced within the iron core by the changing current in the windings. However, the theory shows that the voltage across the primary and secondary windings is proportional to the number of turns, or windings, on each:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

or

$$V_1 = \left(\frac{N_1}{N_2}\right) V_2$$

where

N_1 = secondary winding turns.
N_2 = primary winding turns.

The voltage, or forcelike quantity, at the source clearly is multiplied by the ratio of turns N_1/N_2 to give the voltage at the load. This ratio then must be the M_I of the transformer, as summarized in Equation 12.

$$M_I = \frac{N_1}{N_2} \qquad (12)$$

Use of Equation 12 is exemplified in Example L.

The electrical transformer can step up (increase) or step down (decrease) the primary, or source, voltage, depending upon the ratio of secondary-to-primary turns of the windings. This formula is correct for ideal transformers. Actual transformers waste some energy through resistance heating in the windings, production of eddy currents, and hysteresis losses. All these losses are analogous to friction in the mechanical system.

Because in the ideal case, power input = power output, or $V_2 I_2 = V_1 I_1$, a step-up of voltage leads to a step-down of current, and vice versa. Thus, an electrical transformer also can serve

as a current transformer. It can be used, for example, to produce a large current from a smaller one at the expense of voltage. (See Examples L and M.)

EXAMPLE L: VOLTAGE STEP-DOWN IN A TRANSFORMER

Given: An electrical transformer has 200 turns on the primary and 20 turns on the secondary, and a source voltage is 120 volts.

Find: Voltage available at the load.

Solution:

$$V_1 = (N_1)\, V_2$$

$$= \left(\frac{N_1}{N_2}\right) V_2$$

$$= (20/200)\ 120 \text{ volts}$$

$$V_1 = 12 \text{ volts}$$

EXAMPLE M: STEP-UP OF CURRENT IN A TRANSFORMER

Given: The input current to a transformer is 1 amp at 2000 volts.

Find: The output current at 100 volts, assuming the efficiency is 100%.

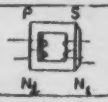
Solution: In an ideal transformer, power input = power output. Therefore —

$$I_2V_2 = I_1V_1$$

$$(1 \text{ amp})(2000 \text{ volts}) = (I_1)(100 \text{ volts})$$

$$I_1 = 20 \text{ amps}$$

Table 1 summarizes the cases for voltage and current multiplication.

TABLE 1. VOLTAGE AND CURRENT RELATIONSHIPS IN A TRANSFORMER				
Number of Turns		Voltage	Current	M_i (Voltage Multiplication)
	$N_1 < N_2$	$V_1 < V_2$	$I_1 > I_2$	$N_1/N_2 < 1$
	$N_1 > N_2$	$V_1 > V_2$	$I_1 < I_2$	$N_1/N_2 > 1$

SUMMARY

Now that a number of force transformers, or simple machines, have been considered in detail, to summarize their major characteristics and emphasize their basic unity may be helpful.

The force transformer is a coupling device between an energy source and an energy load. The transformer changes, or transforms, input values of forcelike, displacementlike, and ratelike quantities into different values at the output. There is, however, a fundamental law governing these changes. The law states that where no energy is lost through frictional effects (ideal transformer), the energy input equals the energy output. Mathematically speaking, the product of forcelike and displacementlike quantities has the same value at input and output. Furthermore, where the transformer operates continuously so that one can speak of rate of work, or power, one can state that in the ideal case, the power input equals the power output. Equivalently, the product of forcelike and ratelike quantities has the same value at input and at output.

In the actual case, some of the energy or power input is used up by frictional affect, so that the energy and power output is less than the corresponding input, as is summarized in Figure 16.

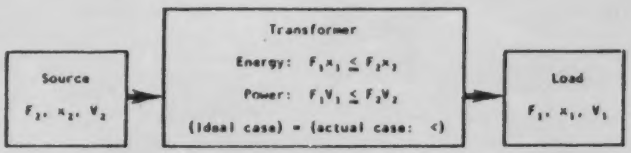


FIG. 16

A characteristic of the force transformer is that where a force advantage is achieved ($F_1 > F_2$), it is achieved at the expense of a distance or velocity disadvantage ($x_1 < x_2$ or $V_1 < V_2$). Only in this way can energy and power products be the same at input and output. The ratio F_1/F_2 — or alternately, the ratio x_2/x_1 — is called the "mechanical advantage" of the transformer. Clearly, the "advantage" is one of force, as long as the M_i is greater than 1.

The foregoing statements apply to all energy systems, as indicated in Table 2, which includes all fundamental energy systems except the thermal system. A few minutes spent studying this table should reinforce the analogous nature of the force transformer in these different energy systems. The table also lists, for the sake of convenience, the M_j for each transformer.

TABLE 2. TRANSFORMERS FOR DIFFERENT ENERGY SYSTEMS

System	Typical Transformers	M_j (ideal)	Forcelike, Displacementlike, Ratelike Quantities	Energy and Power Relationships (= for ideal) (< for actual)
Mechanical translational	Pulley Lever Inclined plane	No. of supporting strand ℓ_2/ℓ_1 (lever arms) Length/rise	Source: F_2, x_2, V_2 Load: F_1, x_1, V_1	Energy: $F_1 x_1 \leq F_2 x_2$ Power: $F_1 V_1 \leq F_2 V_2$
Mechanical rotational	Screw Wheel and axle	$2\pi r_2$/pitch r_2/r_1 (radii)	Source: $\tau_2, \theta_2, \omega_2$ Load: $\tau_2, \theta_1, \omega_1$	Energy: $\tau_1 \Delta\theta_1 \leq \tau_2 \Delta\theta_2$ Power: $\tau_1 \omega_1 \leq \tau_2 \omega_2$
Fluid	Hydraulic jack	A_1/A_2 (areas)	Source: p_2, V_2, V_2 Load: p_1, V_1, V_1	Energy: $p_1 \Delta V_1 \leq p_2 \Delta V_2$ Power: $p_1 V_1 \leq p_2 V_2$
Electrical	Voltage Transformer	N_1/N_2 (turns)	Source: $V_2, Q_2, \dot{I}_2$ Load: V_2, Q_1, I_1	Energy: $V_1 Q_1 \leq V_2 Q_2$ Power: $V_1 I_1 \leq V_2 I_2$

MECHANICAL ADVANTAGE OF A LEVER

MATERIALS

Set of weights up to about 250 grams.
Meter stick prepared with holes drilled to accommodate a large nail, every 10 cm.
Ring stand and clamps.
Assortment of spring balances.

PROCEDURE

Three basic types of levers will be tested in this demonstration. In each case, the lever will be a meter stick having holes drilled every 10 centimeters along its length. Holes will be large enough to accommodate a nail that can act as a pivot. The sharp end of the nail can be clamped onto a ringstand, as shown in Figure 17. In each of the three cases outlined on the following page, hang an arbitrary load with spring or weight holder at an arbitrary point, and then determine experimentally the source force required to produce balance. The required balancing force is applied either by hanging weights or by a spring balance. Do not neglect the weight holder, if one is used. The diagrams below will make each of the three cases clear. Notice that the cases differ in the relative positions of load (L), source (S), and pivot or fulcrum (F). Complete Data Table 1, which includes the data and values calculated from the data.

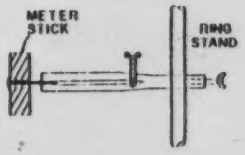
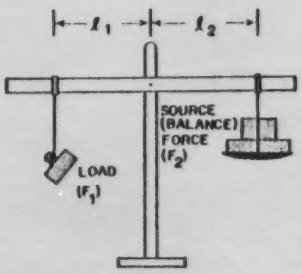
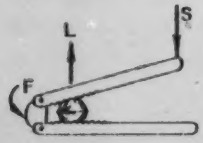


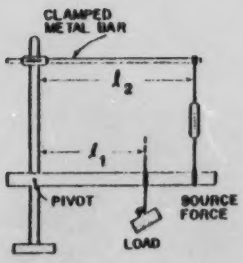
FIG. 17

Case 1: Wooden-beam Lever

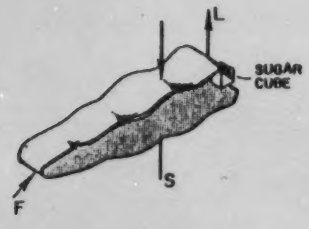


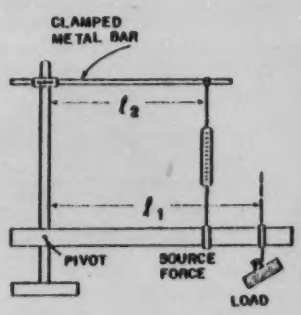
Case 2: Nut-cracker Lever





Case 3: Chemical-tongs Lever





DATA TABLE 1.							
	Source Lever Arm ℓ_2 (cm)	Load Lever Arm ℓ_1 (cm)	Load Force F_1 (dynes)	Source Force F_2 (dynes)	M_i ideal ℓ_2/ℓ_1	M_a actual F_1/F_2	$\eta = \frac{M_a}{M_i}$ (100)
Case 1							
Case 2							
Case 3							

MECHANICAL ADVANTAGE OF AN INCLINED PLANE

MATERIALS

Smooth board about 1 meter long.
Small car that can carry various weights.
Fixed pulley that can be clamped to the end of a board.

PROCEDURE

Determine the weight (source force) needed just to roll the car (load) up the incline, as shown in Figure 18. Make several trials with different weights in the car. Add weights gradually to weight holder until car just rolls up the incline without acceleration. Add the weight of the car to the weight of the load it carries. Measure the incline length and rise, and complete Data Table 2.

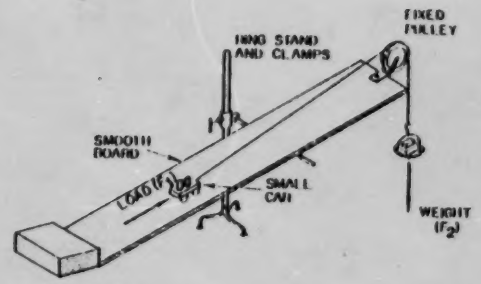


FIG. 18

DATA TABLE 2.

Length of Incline (cm)	Rise of Incline (cm)	Load (dynes)	Source Force (dynes)	M_i $\frac{\text{length}}{\text{rise}}$	M_a F_1/F_2	Efficiency

ELECTRIC TRANSFORMER EFFICIENCY

MATERIALS

12-V transformer.
a.c. Milliammeter 0-100 MA.
a.c. Ammeter 0-1 A.
Volt-ohm-milliammeter.
Knife switch.
Clip leads
a.c. power cord.
20 Ω, 10w resistor.

PROCEDURE

1. Construct the circuit shown in Figure 19.

2. Plug in a.c. power cord. Be careful to avoid any bare wires or contacts while the cord is plugged into the outlet.

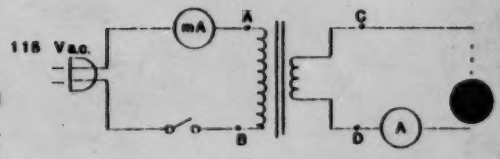


FIG. 19

3. Connect VOM to measure a.c. voltage between points A and B of Figure 19.

4. Close switch. Measure and record in Data Table 3 the values for transformer input voltage and current. Open switch.

5. Connect VOM to measure a.c. voltage between points C and D of Figure 19.

6. Close switch. Measure and record values for transformer output voltage and current. Open switch and unplug power cord.

7. Complete Data Table 3.

DATA TABLE 3.	
Input	Output
Voltage	
Current	
Power	
Transformer efficiency = _____	

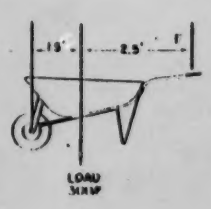
SELF CHECK

1. Describe specific force transformers in the mechanical translational, mechanical rotational, fluidal, and electrical systems; and discuss their fundamental similarity as transformers of forcelike quantities.

2. Define:

   Ideal mechanical advantage.
   Actual mechanical advantage.
   Efficiency.

3. A wheelbarrow contains 300 lb of sand. If the distance from the center of the load to the wheel is 1.5 ft, and the distance from the handle to the wheel is 4 ft, what lifting force F must be applied at the handle to move the sand?



4. The total weight of skiers and equipment on a ski tow is 3000 lb. The cable of the ski tow pulls this load at an incline of 25°. What is the tension in the cable if frictional forces are negligible?

5. A wheel and axle is used to lift a 2400-lb load. The wheel diameter is 60 inches, and the axle diameter is 5 inches. The operator must exert a force of 250 lb to lift the load. Determine:

   Ideal mechanical advantage (M_i).
   Actual mechanical advantage (M_a).
   Efficiency of the transformer.

6. A screw jack is being used to lift a load of 2 tons. The screw pitch is 0.25 inches, and handle length 20 inches. In the ideal case, what effort is needed at the end of the handle? If the efficiency is 20%, what effort is needed?

7. The two pistons of a hydraulic press have diameters of 4 inches and 64 inches. What force must be applied by the smaller piston to give a compressive force of 50 tons at the larger piston? How far does the small piston travel in moving the large piston 1 inch?

8. A doorbell transformer has primary windings of 720 turns and secondary windings of 48 turns. The source voltage is a 120-volt house circuit. What voltage is delivered to operate the bell?

9. Discuss how the role of resistance in a transformer dissipates energy input and reduces efficiency.

10. Describe the power input and the power output of a transformer that operates continuously.

REFERENCES

Beiser, Arthur. Modern Technical Physics. 3rd ed. Menlo Park: CA: The Benjamin/Cummings Publishing Company, Inc., 1979.

Dierauf, Edward J., Jr., and Court, James E. Unified Concepts in Applied Physics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1979.

Faber, Rodney B. Applied Electricity and Electronics for Technology. New York: John Wiley & Sons, 1978.

Halliday, David and Resnick, Robert. Fundamentals of Physics. Revised Printing. New York: John Wiley & Sons, Inc., 1974.

Harris, Norman C. and Hemmerling, Edwin M. Introductory Applied Physics. 3rd ed. New York: McGraw-Hill Book Company, Inc., 1972.

Larson, Boyd. Transistor Fundamentals and Servicing. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1974.

McNickle, L. S., Jr. Simplified Hydraulics. New York: McGraw-Hill Book Company, 1966.

Sears, Francis W.; Zemansky, Mark W.; and Young, Hugh D. College Physics. 4th ed. Reading, MA: Addison-Wesley Publishing Company, 1974.

Shortley, George and Williams, Dudley. Elements of Physics. 5th ed. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1971.

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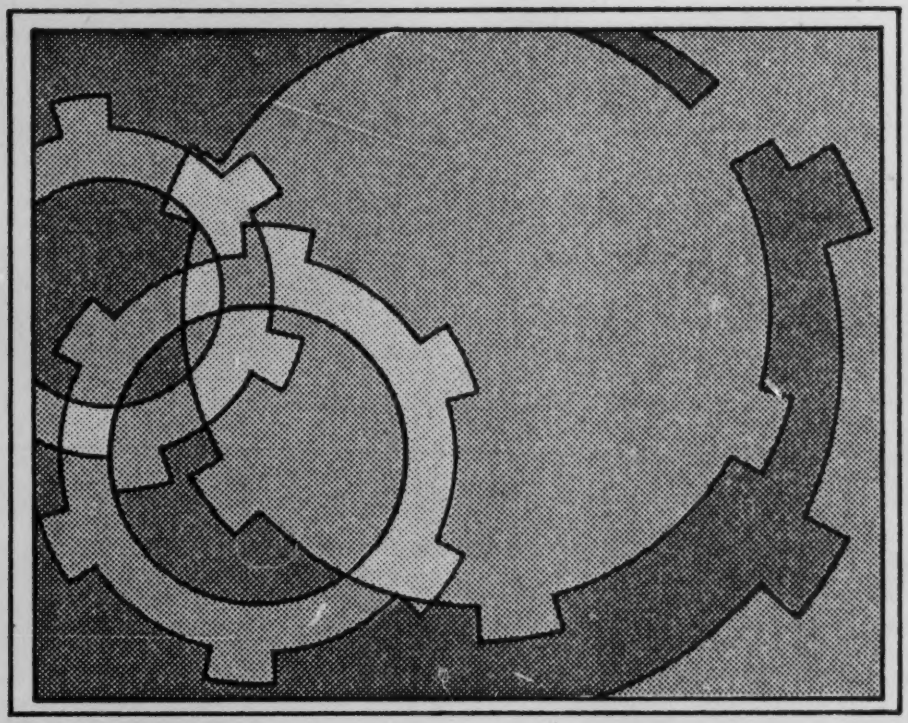
UNIFIED TECHNICAL CONCEPTS

MODULE 8 M 1

LEVERS



MECHANICAL APPLICATION OF FORCE TRANSFORMERS

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

The lever is one of the simplest and most common mechanical force transformers. Depending upon its configuration, the lever may be used to amplify either force or displacement, or to change their directions.

In this module, the student will investigate the lever as a force transformer. In the laboratory, the student will measure and calculate the mechanical advantages of several levers.

PREREQUISITES

Student should have completed Concept Module 8-0, "Force Transformers," and Application Module IM3, "Addition of Torques."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate the fourth quality, given three of the following ones:
  - Load force.
  - Applied force.
  - Lever arm of load force.
  - Lever arm of applied force.
- Given the arm lengths of a lever, calculate its ideal mechanical advantage (M_i).
- Given the load force and applied force of a lever, calculate the actual mechanical advantage (M_a).
- Determine actual mechanical advantage and efficiency of a lever, given its dimensions.
- Sketch and identify the three classes of levers; give two examples each of how they are used.
- Given the appropriate equipment, determine the actual mechanical advantage and the efficiency of a lever.

DISCUSSION

Figure 1 shows the forces applied to a class 1 lever. In this figure, assume that the lever is a rigid rod with no mass and that all forces are applied at right angles to the lever. The force F_1 is the load force and has a lever arm of ℓ_1: F_2 is the applied force and has a lever arm of ℓ_2. The force F_p is the force applied by the pivot, or fulcrum.

In most applications of the lever, the system is very near the equilibrium condition; that is, the sum of both the forces and the torques on the lever is zero:

$$\Sigma F = F_1 + F_2 - F_p = 0$$

$$(\Sigma T)_{pivot} = F_1\ell_1 - F_2\ell_2 = 0$$

FIG. 1

where

ΣF = sum of forces on lever.
ΣT = sum of torques on lever at pivot.
F_1 = load force of lever arm ℓ_1.
F_2 = applied force of lever arm ℓ_2.
F_p = force applied by pivot, or fulcrum.

This equation represents the condition of the lever when there is no motion or when there is motion of the lever at a constant rate.

If the pivot of an ideal lever is assumed to be fixed, the sum of forces must remain zero; however, the sum of torques can be other than zero. If the lever is initially at rest in the equilibrium condition, and if F_2 is increased, F_p increases; and the sum of the forces remains zero. Increased torque $F_2\ell_2$ is not offset, however, by an increase in $F_1\ell_1$ (because neither F_1 nor ℓ_1 changed); and a clockwise angular acceleration takes place. If F_1 is increased later, an opposing angular acceleration is created; and the lever rotates in the opposite direction. If F_2 moves through a vertical distance of x_2, F_1 then moves through a vertical distance of x_1 in the opposite direction. Equation 1 gives the relationships between all quantities for a lever.

$$\frac{F_2}{F_1} = \frac{\ell_1}{\ell_2} = \frac{x_1}{x_2} \qquad (1)$$

where

x_1 = vertical displacement (distance) of load force in one direction.

x_2 = vertical displacement (distance) of applied force in opposite direction to x_1.

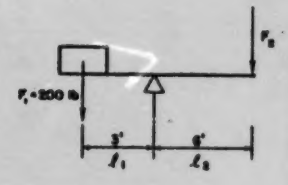
Example A shows use of Equation 1 in solving a problem.

EXAMPLE A: LEVER ARM CALCULATION

Given: The lever shown is used to raise a 200-lb load.

Find: (a) Value of F_2 for equilibrium and (b) height of the load if F_2 moves downward 0.5 feet.

Solution:



(a) $\frac{F_2}{F_1} = \frac{\ell_1}{\ell_2}$ or $F_2 = \frac{F_1 \ell_1}{\ell_2}$

$F_2 = \frac{(200\ \text{lb})(3\ \text{ft})}{(6\ \text{ft})}$

$= \frac{600\ \text{lb-ft}}{6\ \text{ft}}$

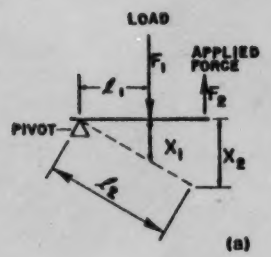
$F_2 = 100\ \text{lb}$

(b) $\frac{x_1}{x_2} = \frac{\ell_1}{\ell_2}$ or $x_1 = \frac{x_2 \ell_1}{\ell_2}$

Therefore,

$x_1 = \frac{(0.5)(3)}{6}$

$x_1 = \frac{1.5}{6} = 0.25\ \text{ft}$



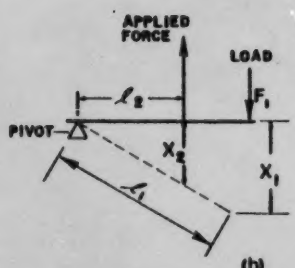


FIG. 2

Figure 1 shows a class I lever, in which the fulcrum always is located between the load force and the applied force. Figure 2 shows two additional classes of levers.

In a class II lever, the fulcrum is at one end of the lever; the applied force is at the other end; and the load is between the two. In this type lever, the load force is always greater than the applied force.

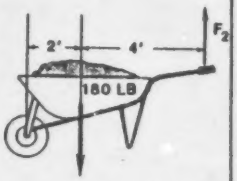
In a class III lever, positions of the load force and applied force are exchanged. The applied force always is greater than the load force.

Equation 1 applies to all classes of levers. Examples B and C show the use of Equation 1 in solving additional problems.

EXAMPLE B: WHEELBARROW LOAD CALCULATION

Given: A loaded wheelbarrow weighs 180 lb and its center of gravity is 2 ft from the wheel axle. The lifting force is applied 4 feet farther from the wheel axle.

Find: (a) Applied force and (b) how far the load rises when the applied force rises 15 inches.



Solution:

(a) $F_2 = \frac{F_1 \ell_1}{\ell_2} = \frac{(180\ \text{lb})(2\ \text{ft})}{6\ \text{ft}}$

$F_2 = 60$ lb

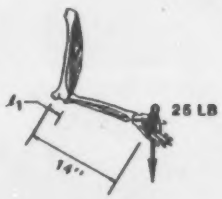
(b) $x_1 = \frac{x_2 \ell_2}{\ell_2} = \frac{(15\ \text{in})(2\ \text{ft})}{(6\ \text{ft})} = \frac{(15\ \text{in})(24\ \text{in})}{(72\ \text{in})}$

$x_1 = 5$ in

EXAMPLE C: HUMAN ANATOMY STRENGTH CALCULATION

Given: The length of a person's arm from elbow to palm is 14 inches. When the biceps are shortened by 1/4 inch, the 25-lb weight in the hand is raised by 1 3/4".

Find: (a) Tension in the person's muscle and (b) lever arm of the applied force.



Solution:

(a) $F_2 = \frac{F_1 x_1}{x_2} = \frac{(25\ lb)(1.75\ in)}{(0.25\ in)}$

$F_2 = 175\ lb$

(b) $\ell_1 = \frac{\ell_2 x_2}{x_1} = \frac{(14\ in)(0.25\ in)}{(1.75\ in)}$

$\ell_1 = 2\ in$

Mechanical Advantage for an ideal lever is given by Equation 2.

$$M_I = \frac{\ell_2}{\ell_1} \qquad (2)$$

where

M_I = ideal mechanical advantage.
ℓ_1 = lever arm of load force.
ℓ_2 = lever arm of applied force.

In the ideal case, the assumption often is made that the lever itself has no mass and that there is no friction at the pivot point. Because this assumption never really is the case, we usually find that the actual mechanical advantage usually is found to be different from the ideal. The actual mechanical advantage is defined by Equation 3.

$$M_a = \frac{F_1}{F_2} \qquad (3)$$

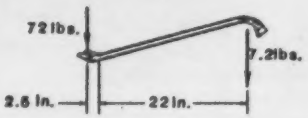
where

M_a = actual mechanical advantage.
F_1 = load force.
F_2 = applied force.

Example D shows the use of Equations 2 and 3 in solving a problem.

EXAMPLE D: LOAD LIFTING USING A CROWBAR

Given: A crowbar used to lift a 72-lb load vertically with an applied force of 7.2 lb. The respective lever arms are 2.5 in and 22 in.



Find: Efficiency of the crowbar.

Solution:

$$M_I = \frac{\ell_2}{\ell_1} = \frac{22\ \text{in}}{2.5\ \text{in}} = 8.8$$

$$M_a = \frac{F_1}{F_2} = \frac{72\ \text{lb}}{7.2\ \text{lb}} = 10$$

$$\% \text{ efficiency} = \frac{M_a}{M_I} \times 100 = \left(\frac{10}{8.8}\right)(100) = 114\%$$

Notice that, in this example, the efficiency is greater than 100%, the reason being that the weight of the heavy metal bar has been neglected. The downward gravitational force acting at the center of gravity of the crowbar actually assists the applied lifting force.

APPLICATION OF LEVERS

Class I levers are employed when a change in direction of a force is necessary. They normally are used to amplify forces, as in the crowbar; but they also can be used to amplify displacement. Class II levers are used to amplify a force without changing its direction. A nutcracker is an example of this type of lever. A class III lever amplifies displacement without changing the direction of the force. The skeletal system of the human body makes extensive use of class III levers. (See Figure 3.)

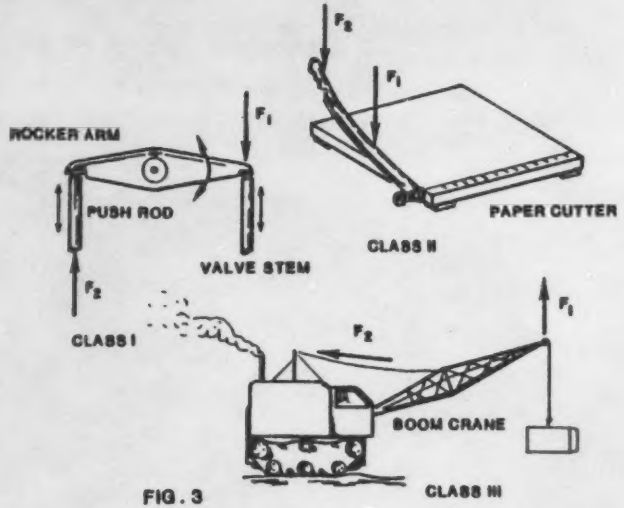


FIG. 3

SUMMARY

The lever is a common, simple force transformer that can amplify force or displacement and can change their directions. Class I levers change direction of force and amplify it. Class II levers amplify force without changing its direction. Class III levers amplify displacement without changing direction of force.

MECHANICAL ADVANTAGE OF LEVERS

CLASS I LEVER

MATERIALS

Eight-inch piece of 2 inch x 4 inch wood.
Two-foot piece of 1 inch x 8 inch wood.
Spring balance with 50-lb capacity.
Crowbar.

PROCEDURE

Note: All loads can consist of weights or sandbags of same weight as measured with bathroom scales.

1. Position wooden block, board, crowbar and spring balance as shown in Figure 4.

2. Have a classmate stand in center of board; apply tension to spring balance until board is level. Record values of ℓ_1, ℓ_2 and F_2 in Data Table.

3. Weigh classmate; record one half weight of classmate as F_1 in Data Table.

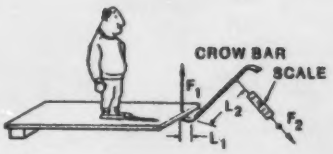


FIG. 4

CLASS II LEVER

MATERIALS

Bathroom scales.
Set of English weights or sandbags.
1/2-inch diameter steel bar 4 ft long.

PROCEDURE

1. Hang a weight of 40 to 60 lb at approximate center of steel bar, as shown in Figure 5. Have classmate stand on bathroom scales, and hold one end of bar while other end rests on a table.

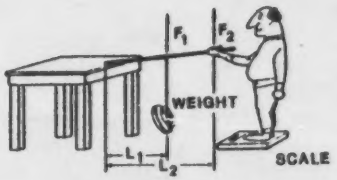


FIG. 5

2. Measure and record ℓ_1, ℓ_2 and reading from bathroom scales.

3. Record F_1 as weight hung on bar and F_2 as difference between bathroom scale reading and classmate's weight.

CLASS III LEVER

MATERIALS

Set of English weights or sandbags.
Spring balance with 50-lb capacity.
1/2-inch diameter steel bar 4 ft long.

PROCEDURE

1. Hang a 10-lb weight from end of steel bar, as shown in Figure 6. Place other end of bar under top of a heavy table; support bar about one third of way from pivot with spring balance.

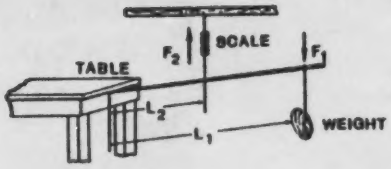


FIG. 6

2. Measure ℓ_1, ℓ_2, F_1 and F_2; record them in Data Table. Perform all necessary calculations; complete Data Table.

3. Explain any differences in % efficiency between the three classes of levers tested.

DATA TABLE							
Class	ℓ_1 (in)	ℓ_2 (in)	F_1 (lb)	F_2 (lb)	M_i	M_a	% efficiency
I							
II							
III							

SELF CHECK

1. A 95-lb boy sits on a seesaw 6 feet from the pivot. How far from the pivot must a 85-lb girl sit to balance him?

2. Draw sketches to identify the three classes of levers; list two examples of each class.

3. In a nutcracker (class II lever), a force of 32 lb is applied 6.5 inch from the fulcrum. What is the force on a nut 1.25 inch from the fulcrum?

4. In a machine for throwing clay targets, a force of 210 lb is applied to a lever 4 inch from the fulcrum. What force is experienced by the clay target 24 inch from the fulcrum?

5. A class II lever is used as an input force transformer to drive a hydraulic jack. An applied force of 50 lb produces a downward force of 600 lb on the hydraulic cylinder. The lever arms are 1.5 inch and 19 inch. Find (a) ideal mechanical advantage, (b) actual mechanical advantage and (c) percentage efficiency of the lever.

REFERENCES

Beiser, Arthur. Modern Technical Physics. Third Edition. Menlo Park, CA: The Benjamin/Cummings Publishing Company, Inc., 1979.

Harris, Norman C. and Hemmerling. Introductory Applied Physics. Third Edition. New York: McGraw-Hill Book Company, Inc., 1972.

Sears, F. W. and Zemansky, M. W. Modern College Physics, Part I: Mechanics and Thermodynamics. Reading, MA: Addison-Wesley Publishing Company, 1962.

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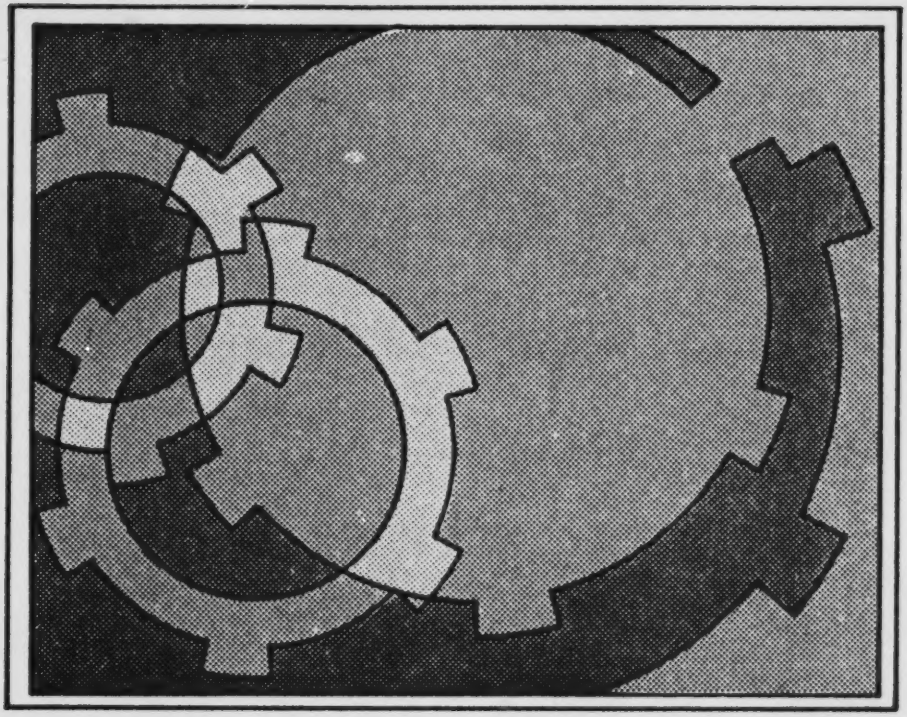
UNIFIED TECHNICAL CONCEPTS

MODULE 8 M 2

WEDGES AND INCLINED PLANES



MECHANICAL APPLICATION OF

FORCE TRANSFORMERS

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

A wedge is a device that transforms a small force applied in one direction into a pair of larger opposing forces in a direction almost perpendicular to the original force. A wedge consists of two inclined planes joined together to form an isosceles triangle and is employed to separate two objects from one another. Applications of the wedge range from the blade of an axe used to split a log to an ice ship cutting through icy water.

In this module the student will investigate the wedge as a pair of inclined planes. The mechanical advantage of a wedge will be calculated and measured in the laboratory and the efficiency of the wedge will be determined.

PREREQUISITES

Student should have completed Concept Module 8-0, "Force Transformers," and Application Module IM2, "Addition of Mechanical Forces."

OBJECTIVES

To complete this module, the student should be able to:

- Calculate forces present when a wedge of known angle is driven into an object by a known force.
- Calculate the theoretical mechanical advantage of a wedge of known angle.
- Determine experimentally the mechanical advantage of a wedge.
- Given theoretical and actual mechanical advantages of a wedge, determine its efficiency.

DISCUSSION

Figure 1 shows a wedge. It is composed of two identical inclined planes (1 and 2) back to back. The wedge angle (2 θ) is twice the angle (θ) of one of the inclined planes.

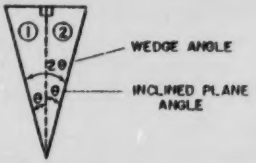


Fig. 1

Figure 2 shows the forces present on a wedge. The force F_O is the total downward force on the wedge. It consists of the weight of the wedge plus the driving force applied to the wedge. F_f is an opposing upward force and is the result of friction between the wedge and the material it encounters. Because the wedge and the forces acting on it are symmetrical about the center line, only one of the two inclined planes will be considered. The downward force F_Y on one of the inclined planes is half of F_O. This produces a normal force F which has F_y as a vertical component such that $F_y = F \sin \theta$. The horizontal component this normal force is F_x. This is the force that does the splitting work of the wedge. It is given by:

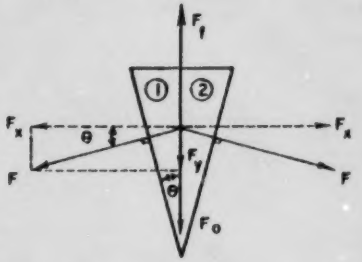


Fig. 2

$$F_x = F \cos \theta = \left(\frac{F_y}{\sin \theta}\right) \cos \theta = \frac{F_y}{\tan \theta}.$$

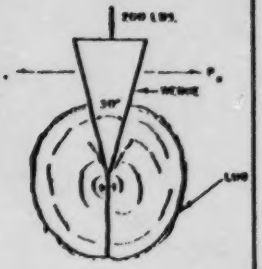
The total "splitting" force of the wedge is $2 F_x$ so that $2 F_x = 2 F_y / \tan \theta = F_O / \tan \theta$. This result is summarized in Equation 1.

$$\text{Splitting force} = \frac{\text{downward force on wedge}}{\text{tangent of half the wedge angle}}.$$

$$\text{In symbols:} \quad 2 F_x = \frac{F_O}{\tan \theta}. \qquad (1)$$

Example A shows the use of this equation in solving a problem.

In Example A, a vertical force of 200 lb on the wedge results in a splitting force of 746 lb on the log. The ratio of these two numbers is the mechanical advantage of the wedge as given in Equation 2.



EXAMPLE A: SPLITTING A LOG

Given: A wedge with a wedge angle of 30° used to split a log. A force of 200 lb is delivered on top of the wedge.

Find: Total splitting force applied to the log. Neglect the weight of the wedge compared to 200 lb.

Solution: The angle (θ) of each inclined plane is $\frac{30°}{2} = 15°$.

Total splitting force is:

$$\Sigma F_x = \frac{F_o}{\tan \theta}$$

$$= \frac{200 \text{ lb}}{\tan (15°)}$$

$$= \frac{200 \text{ lb}}{0.268}$$

$$\Sigma F_x = 746 \text{ lb.}$$

$$\text{Actual mechanical advantage} = \frac{\text{splitting force applied to work piece by wedge}}{\text{vertical force applied to wedge}}.$$

$$\text{In symbols: } MA = \frac{2 F_x}{F_o}. \qquad (2)$$

(Actual MA is based on measured forces.)

By substituting $2 F_x = F_o/\tan \theta$ into Equation 2 and simplifying, we get:

$$MA = \frac{2 F_x}{F_o} = \frac{F_o/\tan \theta}{F_o} = \frac{1}{\tan \theta} = \frac{2h}{b}.$$

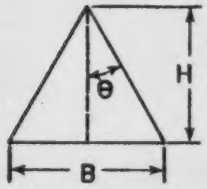
This is summarized in Equation 3.

$$\text{Theoretical mechanical advantage} = \frac{1}{\tan \theta}$$

$$= 2\,\frac{h}{b}$$

$$= 2\,\frac{\text{wedge height}}{\text{wedge base}}$$

$$\text{Theoretical mechanical advantage} = \frac{1}{\text{tangent of inclined plane angle}} \quad (3)$$



Theoretical MA is based on wedge geometry.

In most practical applications of the wedge, frictional forces are high. Equation 3 may be thought of as giving the theoretical mechanical advantage of a wedge because it is based only on the wedge geometry. Equation 2 will give actual mechanical advantage if the forces involved are the measured forces. Efficiency of the wedge depends on the frictional forces present and is given by Equation 4.

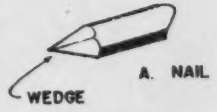
$$\text{Efficiency} = \frac{\text{actual mechanical advantage}}{\text{theoretical mechanical advantage}}.$$

$$\text{In symbols:} \quad \text{Eff} = \frac{\text{MA (actual)}}{\text{MA (theoretical)}}. \quad (4)$$

Example 8 shows the use of Equations 2, 3 and 4 in solving a problem.

A wedge is used to separate two objects or split a single object into two parts. Obvious applications include cutting tools such as the axe and the knife. Less obvious applications include the wedgelike bow streamlining that allows higher speeds for ships through water and wedge fuselage of aircraft. In these cases the wedge allows a small forward force to produce a larger splitting force to move aside ice, water or air.

Piercing tools also employ the wedge. Figure 3a shows the point of a nail. It is usually composed of two wedges at right angles to one another. Figure 3b shows a wood screw. It consists of a wedge wound in a spiral about a core. Notice that the core-shaped piercing point is another application of the principle of the wedge.



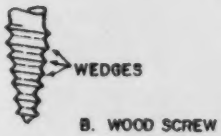
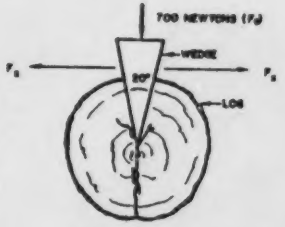


Fig. 3

EXAMPLE B: SPLITTING A LOG WITH A WEDGE

Given: A 20° wedge used to force two surfaces apart. A downward force of 700 N is necessary to produce a total splitting force of 2460 N.

Find: Efficiency of the wedge.



$$\text{Actual MA} = \frac{2F_s}{F_d}$$

$$= \frac{2460\ \text{N}}{700\ \text{N}}$$

$$\text{Actual MA} = 3.51.$$

$$\text{Theoretical MA} = \frac{1}{\tan \theta}$$

$$= \frac{1}{\tan 10^\circ}$$

$$= \frac{1}{0.176}$$

$$\text{Theoretical MA} = 5.67.$$

$$\text{Efficiency} = \frac{\text{MA (actual)}}{\text{MA (theoretical)}} = \frac{3.51}{5.67}$$

$$= 0.62$$

$$\text{Efficiency} = \frac{\text{MA (actual)}}{\text{MA (theoretical)}} = 62\% \text{ efficiency.}$$

This is a high efficiency for this type of a force transformer.

MATERIALS

Two metric spring scales, 5000-g capacity.
Set of metric weights.
Set of three sheet metal wedges: 10°, 20° and 30°.
Triple beam balance.
Two pieces of wood connected with a hinge (described in procedure).
String.

PROCEDURE

1. Bend three sheet metal rectangles into wedges with different angles 2 θ as shown in Figure 4. Weigh each wedge; record its mass in grams.
2. Set up hinged wooden pieces and spring scales as shown in Figure 5. Hinged wooden blocks should rest on table top. Tension in springs should be adjusted to give a reading of near zero on spring scales. Make sure hinge is well-oiled to reduce friction.
3. Separate two pieces of wood slightly; place one metal wedge between them. Place weights on top of wedge until pieces of wood are forced farther apart. Record necessary data in data table.
4. Repeat for other wedges.
5. Complete sample calculations. Perform all calculations necessary to complete results section.

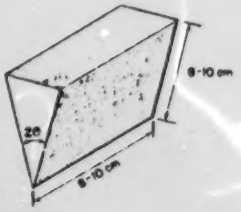


Fig. 4

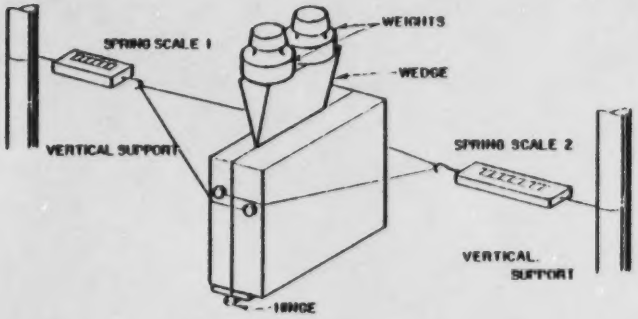


Fig. 5

DATA TABLE				
Wedge Angle 2 θ (Degrees)	Mass of Wedge (g)	Force on Scale 1	Force on Scale 2	Weight on Wedge
a.				
b.				
c.				

Sample calculation: Use row (a) of data table.

Wedge angle: $2\theta =$ _____°.

Inclined plane angle: $\theta =$ _____°.

Theoretical mechanical advantage: $MA = \frac{1}{\tan \theta} = \frac{1}{\tan (____°)}$

$=$ _____.

Downward force: F_o = (mass of wedge + mass of weight) x 980 cm/s²

$=$ (_____ g + _____ g) x 980 cm/s²

$=$ _____ dynes.

Splitting force: $2F_x$ = (force on scale 1 + force on scale 2)

$=$ _____ dynes.

Actual mechanical advantage: $MA = \frac{2F_x}{F_o} = \frac{____ \text{ dynes}}{____ \text{ dynes}}$

$=$ _____.

Percent efficiency = $\frac{\text{actual MA x 100}}{\text{theoretical MA}}$

$=$ _____ ÷ _____ x 100

$=$ _____.

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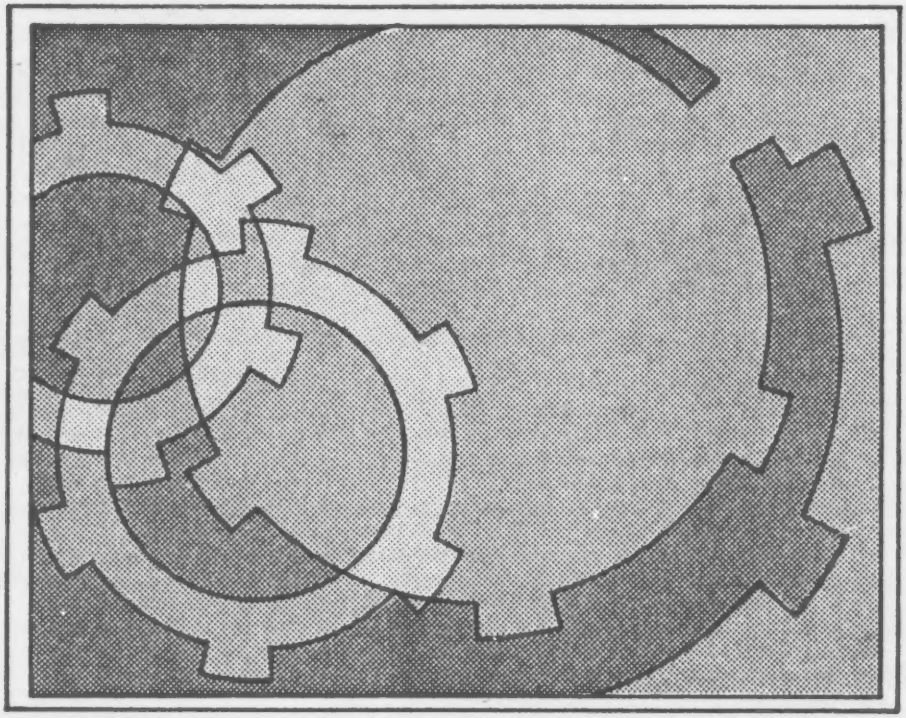
UNIFIED TECHNICAL CONCEPTS

MODULE 8 M 3

DRIVE SYSTEMS



MECHANICAL APPLICATION OF FORCE TRANSFORMERS

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

In rotational mechanical systems, the forcelike quantity is torque. Torque transformers called "drive systems" are employed when changes in torque and angular velocity are desired.

In this module, the student will investigate gear systems, belt drives and disk drives as examples of torque transformers. In the laboratory, the student will examine the disk drive of a turntable.

PREREQUISITES

Student should have completed Concept Module 8-0, "Force Transformers," and Application Module 1M3, "Addition of Torques."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate mechanical advantage in the following systems:
  Gear drive system, given number of teeth on each gear.
  Belt drive system, given radius of each wheel.
  Disk drive system, given radius of each disk.
- Given mechanical advantage of a drive system, input torque and angular velocity, calculate output torque and angular velocity.
- Given the appropriate equipment, determine mechanical advantage, input and output angular velocities and torques of the disk drive of a record player turntable, given necessary data to solve appropriate equations.

Figure 1 shows a torque transformer called a belt drive. During some time t, a point on the belt moves from position A to position B. If there is no slipping, a point on wheel 1 moves through arc A_1B_1 (equal to distance $\overline{AB}$) and a point on wheel 2 moves through an equal arc A_2B_2 (also equal to distance $\overline{AB}$). The angular velocities ω of the two wheels are as follows:

$$\text{since } \theta_1 = \frac{\text{arc } A_1B_1}{r_1}$$

$$\omega_1 = \frac{\theta_1}{t} \qquad \frac{\text{arc } A_1B_1}{r_1 t}$$

and

$$\text{since } \theta_2 = \frac{\text{arc } A_2B_2}{r_2}$$

$$\omega_2 = \frac{\theta_2}{t} = \frac{\text{arc } A_2B_2}{r_2 t}.$$

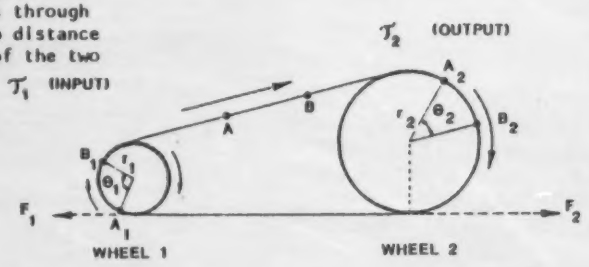


FIG. 1

Since arc A_1B_1 equals arc A_2B_2, dividing one equation by the other results in Equation 1.

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} \qquad (1)$$

where

ω_1 = angular velocity of wheel 1.
ω_2 = angular velocity of wheel 2.
r_1 = radius of wheel 1.
r_2 = radius of wheel 2.

If F_1 and F_2 are tangential forces at the rims of the wheels. torques acting on the two wheels are:

$$\tau_1 = F_1 r_1$$

and

$$\tau_2 = F_2 r_2.$$

F_1 and F_2 each equals the tension in the belt, and they must be equal to each other. After dividing, Equation 2 becomes:

$$\frac{\tau_1}{\tau_2} = \frac{r_1}{r_2} \qquad (2)$$

where

τ_1 = torque of wheel 1.
τ_2 = torque of wheel 2.

Mechanical advantage of the transformer is defined as ratio of the output torque to input torque. Equations 1 and 2 are combined to obtain Equation 3.

$$M = \frac{\tau_2}{\tau_1} = \frac{r_2}{r_1} = \frac{\omega_1}{\omega_2} \qquad (3)$$

where

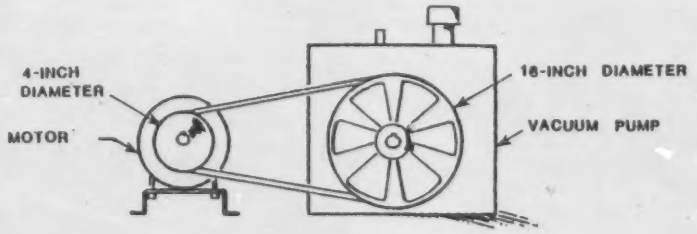
M = mechanical advantage.

An increase in torque results in a corresponding decrease in angular velocity. Example A shows use of Equation 3 in solving a problem.

EXAMPLE A: BELT DRIVE SYSTEM

Given: A belt drive system vacuum pump has a motor wheel with a 4-inch diameter and a 16-inch diameter load wheel. The motor torque is 4 ft·lb, and the motor turns at 480 rpm.

Find: (a) Mechanical advantage of the drive system.
(b) Torque supplied to the load.
(c) Rotational rate of the pump.



Solution: From Equation 3:

(a) $M = \frac{r_2}{r_1}$

$r_1 = \frac{4}{2} = 2$ in

$r_2 = \frac{16}{2} = 8$ in

$M = \frac{8 \text{ in}}{2 \text{ in}} = 4$

(b) $M = \frac{\tau_2}{\tau_1}$

$\tau_2 = M \tau_1$

$\therefore \tau_2 = (4)\ (4 \text{ ft·lb}) = 16 \text{ ft·lb}$

(c) $M = \frac{\omega_1}{\omega_2}$

$\omega_2 = \frac{\omega_1}{M}$

$= \frac{480 \text{ rpm}}{4}$

$\therefore \omega_2 = 120$ rpm

(d) Another solution (from Equation 3) for c would be:

$\frac{r_2}{r_1} = \frac{\omega_1}{\omega_2}$

$\therefore \omega_2 = \omega_1 \frac{r_1}{r_2} = 480\ (\frac{2}{8})$

$\omega_2 = 120$ rpm

Figure 2 shows a disk drive system in which one disk drives another by friction contact. A careful comparison of this diagram with Figure 1 shows that the only difference is the direction of rotation of the second wheel. The disk drive in this figure reverses direction of rotation; a belt drive does not, unless the belt is crossed. All previous equations apply to both disk and belt drives.

Figure 3 shows a gear drive system identical to the disk drive system, except that planetary teeth are cut on the circumference of each disk to create a stronger, more efficient drive, with the number of teeth on each gear proportional to the ra-

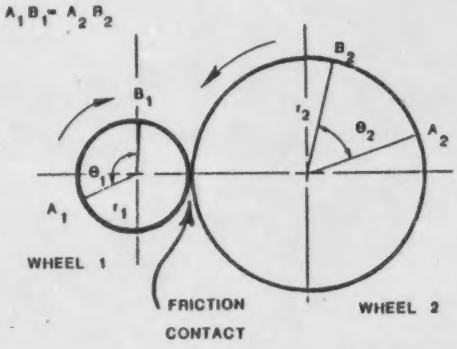


FIG. 2

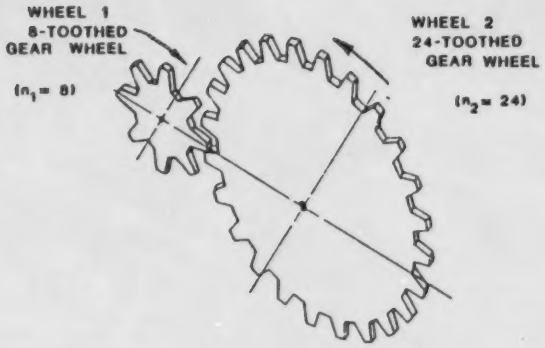


FIG. 3

dius of the gear. Equation 4 is rewritten to obtain Equation 4.

$$M = \frac{\tau_2}{\tau_1} = \frac{n_2}{n_1} = \frac{\omega_1}{\omega_2} \qquad (4)$$

where

n_1 = number of teeth on gear 1.

n_2 = number of teeth on gear 2.

Example B shows use of Equation 4 in solving a problem.

EXAMPLE B: GEAR DRIVE IN CLOCKS

Given: In an alarm clock, a gear with 55 teeth drives a gear with 7 teeth. The first gear has a rotational rate of 3 revolutions per day.

Find: (a) Mechanical advantage of the drive and (b) rotational rate of the second gear.

Solution:

(a) $M = \frac{n_2}{n_1}$

$= \frac{7}{55}$

$M = 0.1273$

(b) $M = \frac{\omega_1}{\omega_2}$

from which

$\omega_2 = \frac{\omega_1}{M}$

$= \frac{3 \text{ r/day}}{0.1273}$

$\omega_2 = 23.57$ r/day

APPLICATIONS OF DRIVE SYSTEMS

Since rotational motion is a convenient method of transferring mechanical power from one point to another, drive systems are normally used with electric motors and internal combustion engines. Gear drives are common in clocks, engine lathes and automobile transmissions. Belt drives often are employed with electric motors in pumps, blowers and tape recorders. In an automobile, belts drive the fan, water pump and power brakes. Disk drives are common in systems requiring accurate constant angular velocity and low torque such as turntables of stereo record players.

Figure 4 shows a variable-diameter pulley system of a drill press. The wide V-belt rides on two cone, or variable-diameter, pulleys. Each pulley is composed of two conical sections whose separation may be varied. The pulley with the cones closest together has the largest effective diameter.

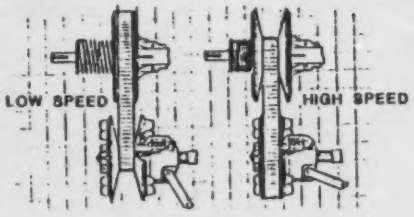


FIG . 4

Figure 5 shows the disk drive system of a record player turntable. The motor shaft with a radius r_1 drives a disk of radius r_2. This disk drives the larger turntable (radius r_3) from the inside. Example C is a problem concerning this figure.

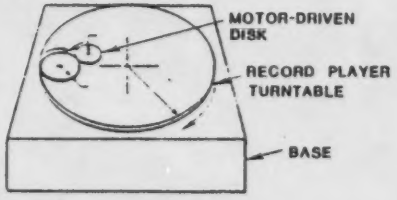


FIG. 5

EXAMPLE C: MULTI DISK DRIVE

Given: Figure 5; the motor rotates at 3600 rpm and $r_1 = 0.785$ mm, r_2 1.85 cm, $r_3 = 8.5$ cm.

Find: Rotational rate of turntable and mechanical advantage of the drive system.

Solution:

$$M\ (r_1 \text{ to } r_2) = \frac{r_2}{r_1}$$

$$= \frac{1.85\ \text{cm}}{0.0785\ \text{cm}}$$

$$= 23.57$$

$$M\ (r_2 \text{ to } r_3) = \frac{r_3}{r_2}$$

$$= \frac{8.5\ \text{cm}}{1.85\ \text{cm}}$$

$$= 4.60$$

System M (r_1 to r_3)

$$= M\ (r_1 \text{ to } r_2) \times M\ (r_2 \text{ to } r_3)$$

$$= (23.57)\ (4.60)$$

$$\therefore \text{ system } M = 108$$

$$\text{Now system } M = \frac{\omega_1}{\omega_3}$$

$$\therefore\ \omega_3 = \frac{\omega_1}{M} = \frac{3600}{108} = 33\text{-}1/3 \text{ rpm}$$

Notice that this is the standard rpm for the long-playing record.

SUMMARY

Drive systems transform torque into a desired angular velocity or an angular velocity into a desired torque. The primary types of drive systems are gear, belt and disk drives. They normally are used with electric motors and/or internal combustion engines.

DISK DRIVE OF A TURNTABLE

MATERIALS

Phonograph turntable.
Micrometer calipers.
Rule (12 in).
Map tack.
Light spring balance (16 oz).

PROCEDURE

1. Remove turntable from its shaft; examine drive mechanism (Figure 6). Note: Turntable is driven from inside by a rubber disk, which is driven by a shaft turned by a motor. The motor shaft has four sections, each of different diameter. If smallest diameter is used, table turns at 16 rpm. If largest diameter is used, table turns at a rate of 78 rpm. The other two sections are for 33-1/3 and 45 rpm. Measure diameter of rubber disk and diameter of 33-1/3 rpm section of motor shaft. Record this data in Data Table.

2. Measure and record inner diameter of turntable.

3. Reassemble turntable.

4. Press tack into turntable surface at a point near rim. Attach spring balance as shown in Figure 7; position it so its line of action is tangent to turntable. Set turntable angular speed for 33-1/3 rpm, and switch on. Record scale reading and value of ℓ in Data Table.

5. Complete calculations in Data Table.

6. Repeat above steps for a second case with an angular speed of 45 rpm.

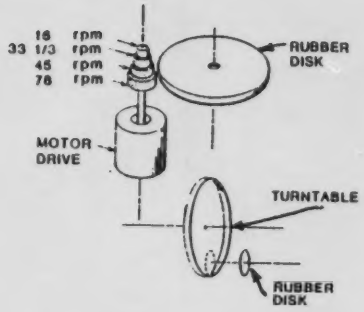


FIG. 6

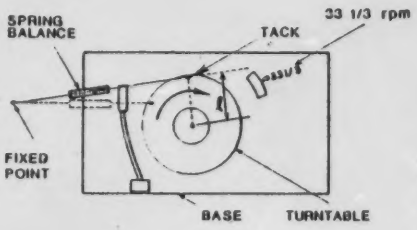


FIG. 7

7. What is the relationship of angular velocity and mechanical advantage to torque?

DATA TABLE

Case 1: $\omega_3 = 33.3$ rpm.

Motor shaft diameter: $d_1 =$ _____ inches.

Disk diamter: $d_2 =$ _____ inches.

Turntable inner diameter: $d_3 =$ _____ inches.

Force on spring balance: $F =$ _____ ounces.

Lever arm of spring balance: $\ell =$ _____ inches.

Mechanical advantage: $M = \left(\frac{d_3}{d_2}\right)\left(\frac{d_2}{d_1}\right)$

$= \frac{d_3}{d_1} =$ _____ inches ÷ _____ inches

$\therefore M =$ _____.

Input angular velocity: $\omega_1 = \omega_3 M$

$\omega_1 = 33.3$ rpm x _____

$\omega_1 =$ _____ rpm.

Output torque: $\tau_3 = F \ell$

$=$ _____ ounces x _____ inches

$\tau_3 =$ _____ ounces x inches.

Input torque: $\tau_1 = \frac{\tau_3}{M} = \tau_3 \div M$

$\tau_1 =$ ounces x inches ÷ _____

$\tau_1 =$ _____ ounces x inches.

Case 2: $\omega_3 = 45.0$ rpm.

$d_1 =$ _____ inches.
$d_2 =$ _____ inches.
$d_3 =$ _____ inches.
} These are the same as measured in Case 1.

$F =$ _____ ounces.

$\ell =$ _____ inches (same as measured in Case 1).

$M =$ _____.

$\omega_1 =$ _____ rpm.

$\tau_3 =$ _____ ounces x inches.

$\tau_1 =$ _____ ounces x inches.

SELF CHECK

1. A gear with 18 teeth drives a gear with 56 teeth. What is the system mechanical advantage?

2. In a belt drive system for a band saw, the motor has a pulley with a diameter of 2-12 inches, and the load has a pulley with a diameter of 14 inches. What is the system mechanical advantage?

3. The disk drive of a turntable has three disks with diameters of 5.75 in, 1.15 in and 0.1065 in. What is the system mechanical advantage?

4. The input torque of the motor in the turntable in problem 3 is 0.42 oz x in and the turntable is rotating at 33-1/3 rpm. Find output torque and angular velocity of the motor.

5. State reasons for differences in ω_1 and τ_1 between Cases 1 and 2 in Data Table.

REFERENCES

Harris, Norman C. and Hemmerling, Edwin M. Introductory Applied Physics. 3rd ed. New York: McGraw-Hill Book Company, 1973.

Sears, Francis W.; Zemansky, Mark W.; and Young, Hugh D. College Physics. 4th ed. Reading, MA: Addision-Wesley Publishing Company, 1974.

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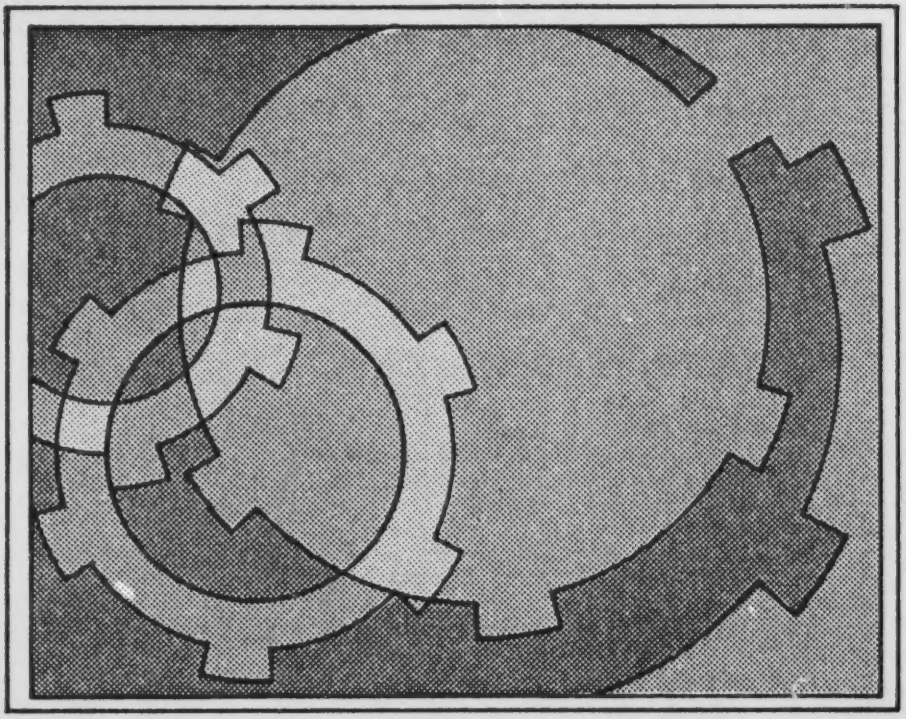
UNIFIED TECHNICAL CONCEPTS

MODULE 8 M 4

GEAR TRAINS



MECHANICAL APPLICATION OF FORCE TRANSFORMERS

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

One of the more important types of force transformers is the gear train. No other force transformation system is used more widely as a means of producing and applying mechanical energy. Gear drive systems commonly are employed to change torque, angular velocity and direction of rotation in rotational mechanical systems. This module discusses the design, characteristics and applications of a variety of common gear drive systems. In the laborabory, the student will examine the gear drive of a metal cutting lathe.

PREREQUISITES

Student should have completed Concept Module 9-0, "Force Transformers," and Application Module 8M3, "Drive Systems."

OBJECTIVES

Upon completion of this module, the student should be able to:

- List three characteristics of rotational motion that can be changed by gear trains.
- Given diagrams of the following gear trains, identify each:

  Simple.
  Compound.
  Reverted.
  Planetary.
  Differential.

- Given the number of gears and teeth on each gear in the following gear trains, determine the mechanical advantage and direction of rotation of the output shaft:

  Simple.
  Compound.
  Reverted.

- Draw and label diagrams of a planetary gear train, showing the direction of rotation of each gear under the following conditions:

Input on sun gear, output on cage, ring gear locked.
Input on sun gear, output on ring gear, cage locked.
Input on ring gear, output on cage, sun gear locked.
Input on cage, output on ring gear, sun gear locked.

- Draw and label a diagram of a simple automobile differential gear train. Show the rotation of each gear during cornering.

- State one application of each of the following gear trains:

  Compound.
  Reverted.
  Planetary.
  Differential.

- Given the gear train of a lathe with a Norton gear box, determine the gear ratio between the works spindle and feed shaft for two different feed speeds.

THE SIMPLE GEAR TRAIN

Figure 1 shows a simple gear drive that consists of two gears. This system transforms rotational motion of the input shaft to output rotational motion with a different angular velocity, torque (forcelike quantity) and direction of rotation. Equation 1 gives the ideal mechanical advantage, also called the gear ratio, of such a gear train.

$$M = \frac{\tau_2}{\tau_1} = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2} \qquad (1)$$

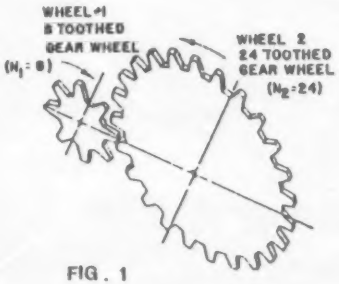


FIG. 1

where

M = mechanical advantage.
τ_1 = torque on gear 1.
τ_2 = torque on gear 2.
N_1 = number of teeth on gear 1.
N_2 = number of teeth on gear 2.
ω_1 = angular velocity of gear 1.
ω_2 = angular velocity of gear 2.

Example A shows the use of Equation 1 in solving a problem.

EXAMPLE A: MECHANICAL ADVANTAGE OF A SIMPLE GEAR TRAIN

Given: In Figure 1, wheel 1 has a rotational rate of 2.4 rad/s and delivers a torque of 0.4 N·m.

Find: (a) Mechanical advantage.
(b) Rotational rate of wheel 2.
(c) Output torque of wheel 2.

Solution: $M = \frac{\tau_2}{\tau_1} = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2}$

(a) $M = \frac{N_2}{N_1}$

$= \frac{24}{8}$

$M = 3$

(b) $\omega_2 = \frac{\omega_1}{M}$

$= \frac{2.4 \text{ rad/s}}{3}$

$= \omega_2 = 0.8 \text{ rad/s}$

(c) $\tau_2 = M\tau_1$

$= (3)(0.4 \text{ N·m})$

$\tau_2 = 1.2 \text{ N·m}$

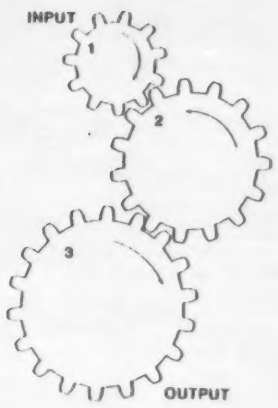


FIG. 2

A simple gear train is one in which the centers of all gears are fixed, and there is only one gear per shaft. Figure 2 is another example of such a gear train. The direction of rotation of the output is the same as that of the input if the train contains an odd number of gears and is opposite in direction for an even number of gears. The mechanical advantage of such a train is determined by applying Equation 1 for each pair of interlocking gears, as shown in Example B.

EXAMPLE B: MECHANICAL ADVANTAGE OF SIMPLE TRAIN WITH MORE THAN TWO GEARS

Given: A four-gear train has gears with gears having the following numbers of teeth: $N_1 = 16$, $N_2 = 8$, $N_3 = 13$, $N_4 = 17$.

Find: Mechanical Advantage.

Solution:

$$M\ (1 \text{ to } 2) = \frac{N_2}{N_1} = \frac{8}{16}$$

$$M\ (2 \text{ to } 3) = \frac{N_3}{N_2} = \frac{13}{8}$$

$$M\ (3 \text{ to } 4) = \frac{N_4}{N_3} = \frac{17}{13}$$

$$M\ (\text{total}) = M\ (1 \text{ to } 2)\ M\ (2 \text{ to } 3)\ M\ (3 \text{ to } 4)$$

$$= \left(\frac{N_2}{N_1}\right)\left(\frac{N_3}{N_2}\right)\left(\frac{N_4}{N_3}\right)$$

$$= \frac{N_4}{N_1}$$

$$M = \frac{17}{16}$$

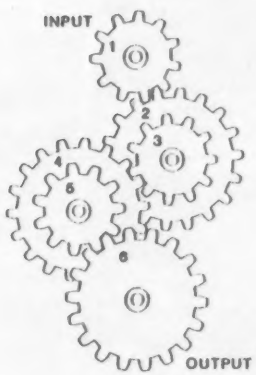


FIG. 3

Example B indicates that in a simple train, the mechanical advantage depends only upon the number of teeth on the input gear and the number on the output gear. Intermediate gears do not effect mechanical advantage.

THE COMPOUND GEAR TRAIN

Figure 3 shows a compound gear train in which some shafts carry two gears that are rigidly connected to rotate at the same rate and with the same torque. The same technique used to calculate the mechanical advantage of a simple gear train can be applied to the compound train, as shown in Example C.

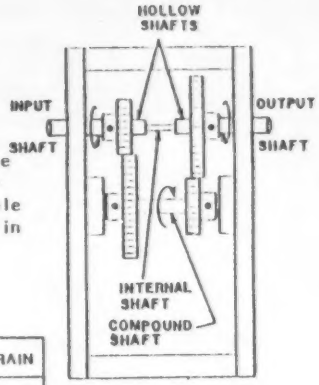


FIG. 4

EXAMPLE C : MECHANICAL ADVANTAGE OF A COMPOUND GEAR TRAIN

Given: Gear train in Figure 3, $N_1 = 11$, $N_2 = 18$, $N_3 = 11$, $N_4 = 18$, $N_5 = 11$, $N_6 = 18$.

Find: Mechanical Advantage.

Solution: The overall mechanical advantage is the product of the mechanical advantage for all gear pairs.

$$M\ (1 \text{ to } 2) = \frac{N_2}{N_1} = \frac{18}{11}$$

M (2 to 3) = 1 because gears are rigidly connected.

$$M\ (3 \text{ to } 4) = \frac{N_4}{N_3} = \frac{18}{11}$$

M (4 to 5) = 1

$$M\ (5 \text{ to } 6) = \frac{N_6}{N_5} = \frac{18}{11}$$

$$M \text{ total} = \left(\frac{N_2}{N_1}\right)\left(\frac{N_4}{N_3}\right)\left(\frac{N_6}{N_5}\right) = \frac{N_2 N_4 N_6}{N_1 N_3 N_5}$$

$$= \frac{(18)(18)(18)}{(11)(11)(11)}$$

$$= 4.38$$

The compound train in Figure 3 is composed of three simple trains — gear 1 drives gear 2, gear 3 drives gear 4, and gear 5 drives gear 6. The mechanical advantage of the compound train is the product of the mechanical advantages of the simple train that it contains.

Compound gear trains are used in mechanical clocks and in drive trains for lathes, power tools, and automobiles. Figure 4 shows a special class of compound train, called a reverted train, in which the input and output shafts are coaxial. This design affords ease of changing gear ratios

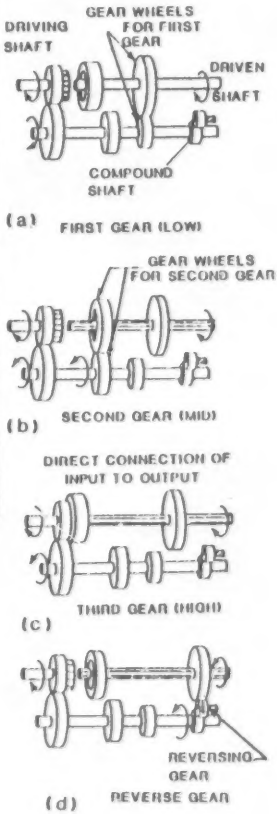


FIG. 5

in a compact space. Its most familiar application is in the automobile transmission in Figure 5.

First gear provides the greatest mechanical advantage to give a high torque and low rotational rate for starting and low-speed operation. Second gear provides a lower mechanical advantage for intermediate speeds. In third gear, the input and output shafts are connected directly to give a mechanical advantage of one for high speed operation. In reverse, an additional gear called an idler is added to reverse the direction of rotation of the output shaft.

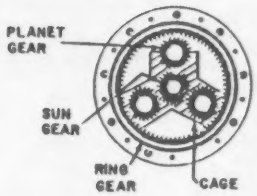


FIG. 6

THE PLANETARY GEAR TRAIN

Figure 6 shows a planetary gear train in which the centers of rotation of some gears are not fixed. This device consists of a central sun gear, one or more planetary gears attached to a carrier called a "cage," and an outer ring gear. The sun gear, cage, and ring gear can be connected to the input or output by concentric shafts. Planetary gears are used as force transformers in bicycles and automatic transmissions for automobiles.

Figure 7 shows how a planetary gear train is used in an automobile. The connections are as follows:

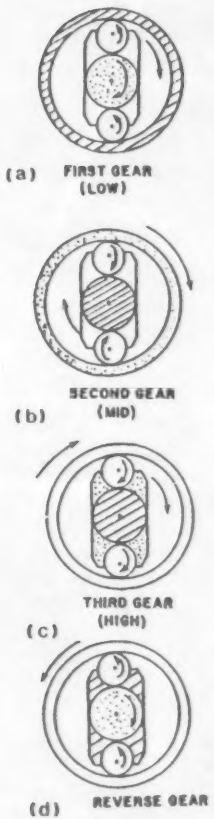


FIG. 7

- Low gear: Sun gear — input.
  Ring gear — locked.
  Cage — output.

  This arrangement gives the greatest mechanical advantage and the lowest speed.

- Mid gear: Sun gear — locked.
  Ring gear — input.
  Cage — output.

  This configuration gives a lower mechanical advantage and a greater speed.

- High gear: Sun gear — locked.
  Ring gear — output.
  Cage — input.

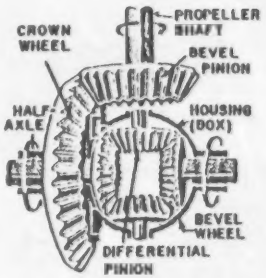
  This connection gives the least mechanical advantage and the greatest speed.

- Reverse gear: Sun gear — input.
  Ring gear — output.
  Cage — locked.

This arrangement reverses the direction of rotation of the output shaft.

The differential gear train in Figure 8 is a special application of the planetary train used to produce two outputs with equal torque, but differing rotational rates.

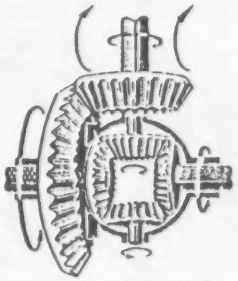
The example shown is the differential train of an automobile. The crown gear is rigidly attached to the housing. Each half axle is attached to a level gear. During straight travel (Figure 8a) the crown gear, housing and both half-axles all rotate at the same rate, and the differential pinions do not rotate with respect to the housing or other gears. During cornering (Figure 8b) one wheel must rotate faster than the other. In this case the pinion gears rotate as shown allowing the two half-axles to rotate at different rates but with equal torques.



(a)

SUMMARY

A simple gear drive consists of two gears that transform rotational motion of the input shaft to output rotational motion having a different angular velocity, torque (forcelike quantity) and direction of rotation. A compound gear train is composed of two or more simple trains; the mechanical advantage (M) of such a train is the product of the trains that it contains. The planetary gear is used for many special purposes because of its small size and unique qualities; for example, it is used in automobile differentials, where it produces two outputs with equal torque but different rotational rates.



(b)

FIG. 8

DETERMINING GEAR RATIOS

MATERIALS

Lathe with Norton gear box (or other suitable gear train).
Tools to remove covers of gear box.
Pencil.

PROCEDURE

1. Disconnect electrical power of lathe and disengage lathe motor.
2. Remove covers to expose gear drive system that connects spindle shaft of lathe to feed shaft (Figure 9).
3. Count number of teeth on each gear and record numbers in data section.
4. Engage movable lever and gear for greatest feed speed as shown in Figure 9.
5. Rotate spindle shaft by hand in direction indicated in Figure 9. Observe direction of rotation of each gear in train and record by drawing appropriate arrows in Figure 9.
6. Calculate mechanical advantage between spindle shaft and feed shaft.
7. Make small index mark on spindle shaft and feed shaft with pencil.
8. Rotate spindle shaft to produce one complete rotation of feed shaft. Count and record rotations of spindle shaft necessary. Estimate fractions of a rotation from final position of index mark on spindle shaft.
9. Repeat procedure for three different feed speeds.
10. Replace all cover plates.

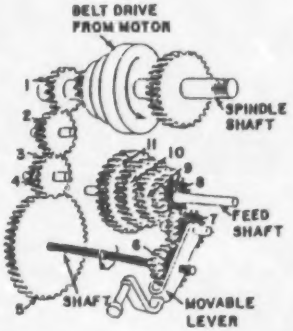


FIG. 9

DATA TABLE					
Gear	Teeth	Gear	Teeth	Gear	Teeth
1		5		9	
2		6		10	
3		7		11	
4		8			

Mechanical advantage (Refer to Figure 9):

$$M\ (1 \text{ to } 8) = \frac{N_3}{N_1} \times \frac{N_5}{N_4} \times \frac{N_8}{N_6}$$

M = _____

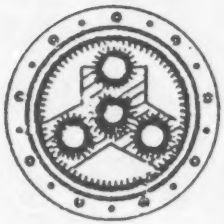
Number of rotations of spindle shaft for one rotation of feed shaft: N = _____

Output Gear	Mechanical Advantage (M)	Rotations (N)
8		
9		
10		
11		

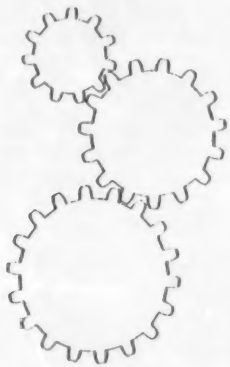
SELF CHECK

1. Gear trains may be used to change what three quantities?

2. Identify each of the following types of gear trains.



(a) _____



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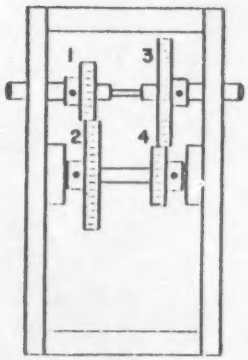
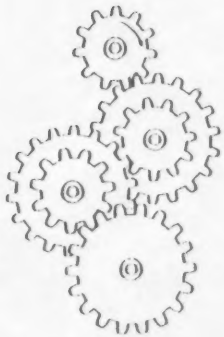


(c) _____

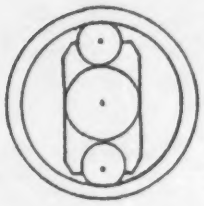


(d) _____

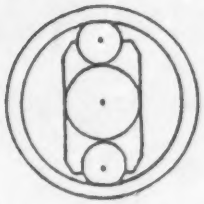
3. In the gear train shown in Problem 2c above, $N_1 = 19$, $N_2 = 26$, $N_3 = 18$, $N_4 = 27$. Find the mechanical advantage.

4. In the gear train shown in Problem 2d above, draw arrows indicating the direction of rotation of each gear.

5. Indicate the direction of rotation of the cage and each gear on the following diagrams. Assume input rotation is clockwise.

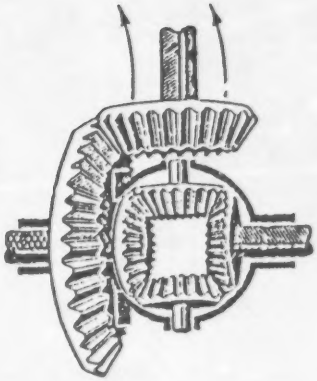


Input on sun gear.
Output on ring gear.
Cage locked.



Input on cage.
Output on ring gear.
Sun gear locked.

6. On the diagram at the right, indicate the direction of rotation of each gear as the automobile turns left.



7. List the type of gear train used in each of the following devices:

   a. Automatic transmission of automobile.
   b. Standard transmission of automobile.
   c. Mechanical clock.

REFERENCES

Beiser, Arthur. Modern Technical Physics. Third Edition. Menlo Park, CA: The Benjamin/Cummings Publishing Company, 1979. See especially Chapter 10.

Technical Education Research Center, ed. "Drives", Electro-mechanical Technology Series, Albany, NY: Delmar Publishers, 1971.

The Way Things Work, Volume 1. New York: Simon and Schuster, 1967.

The Way Things Work, Volume 2. New York: Simon and Schuster, 1971.

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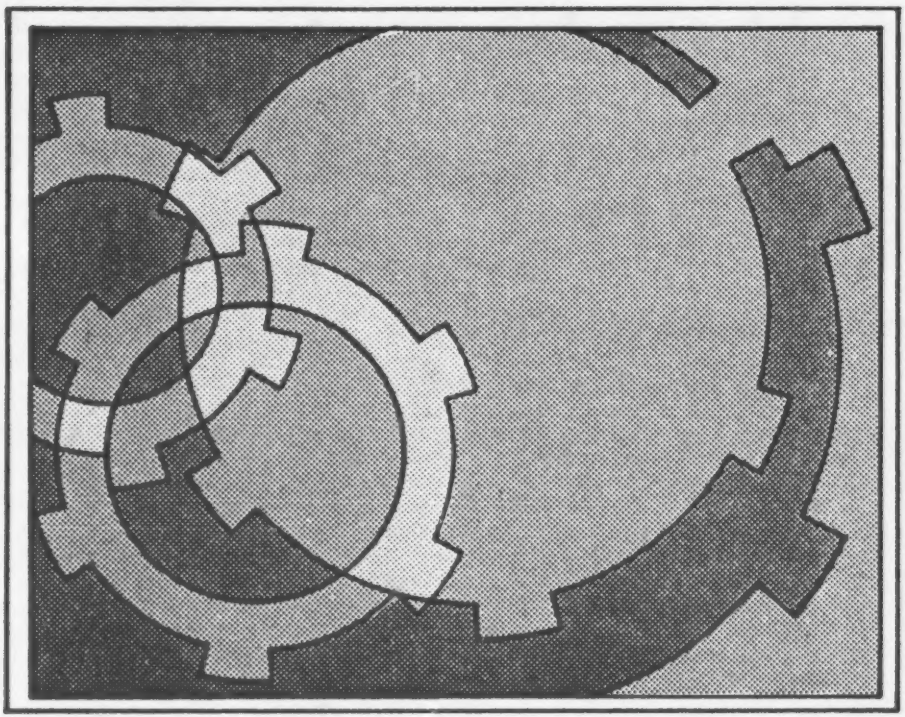
UNIFIED TECHNICAL CONCEPTS

MODULE 8 M 5

BLOCK AND TACKLE



MECHANICAL APPLICATION OF

FORCE TRANSFORMERS

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

The word "machine" conveys many meanings, depending on the context in which it is used. In physics and technology, a very restrictive meaning is adopted. A machine is defined as a "device for the advantageous application of a force," or in other words, a force transformer.

Industrial machines are very complicated devices, but no matter how complex they become, they remain combinations of a few basic simple machines: the lever, inclined plane and hydraulic press.

Among the simple machines that operate on the principle of lever action are pulleys, gears, wheel and axle, block and tackle, and the differential pulley. In this module, various pulley systems, including the block and tackle are presented. In Module 8M6, this discussion will be extended to include the differential hoist.

PREREQUISITES

Student should have completed Concept Modules 8-0, "Force Transformers."

OBJECTIVES

To complete this module, the student should be able to:

- Describe, with the aid of diagrams, various pulley systems; discuss how they act as force transformers.
- Define and/or calculate:
  - Ideal mechanical advantage of a pulley system.
  - Actual mechanical advantage of a pulley system.
  - Efficiency of a pulley system.
- Given appropriate equipment, calculate the mechanical advantage of a pulley system.
- Briefly describe the effect of friction in a pulley system

and explain what effect friction has on the efficiency of the system.

DISCUSSION

A pulley system is most frequently used to lift a large weight through a small distance by applying a small force through a large distance. Thus, the effectiveness of the applied force is magnified, but at the expense of moving a greater distance.

The ratio of the load force (perhaps a weight being lifted) to the applied force is called the actual mechanical advantage. For a given load force, the magnitude of the applied force will always depend upon the kind of machine and the amount of friction.

$$\text{Actual mechanical advantage} = \text{AMA} = \frac{\text{load force}}{\text{applied force}}.$$

$$\text{AMA} = \frac{F_L}{F_A}. \qquad (1)$$

An application of AMA is shown in Example A.

EXAMPLE A: MECHANICAL ADVANTAGE OF A CAR JACK

Given: A downward force of 10 lb exerted on the handle of the car jack. An upward force of 700 lb is exerted by the jack on the car.

Find: Mechanical advantage.

Solution:

$$\text{AMA} = \frac{F_L}{F_A} = \frac{700}{10} = 70$$

Thus, the actual mechanical advantage is 70.

SIMPLE PULLEY SYSTEMS

In many problems it is desirable to find out what the mechanical advantage would be if the friction were zero. Consider a single fixed pulley used to raise a 100-lb load through a vertical distance of 2 ft (Figure 1). In lifting the load, the applied force must act through an equal distance. If friction can be neglected,

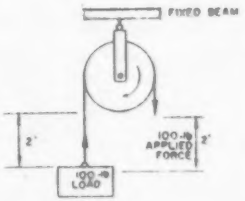


Fig. 1

Input work = output work.

Input (applied) force x distance through which input force acts
$= F_A S_A$ = input work.

Output (load) force x distance through which output force acts
$= F_L S_L$ = output work.

$$F_A S_A = F_L S_L$$

$$F_A = F_L \frac{S_L}{S_A} = 100 \text{ lb} \times \frac{2 \text{ ft}}{2 \text{ ft}}$$

$$F_A = 100 \text{ lb}.$$

If friction is absent, it takes 100 lb to lift 100 lb with a single, fixed pulley. Therefore, it is clear that a single fixed pulley does not transform the force but merely changes the direction of the effort.

In general, the frictionless mechanical advantage is called the ideal mechanical advantage, IMA. The IMA is defined as the ratio of the distance through which the applied (input) force acts to the distance through which the load (output) force acts.

$$\text{Ideal mechanical advantage} = \text{IMA} = \frac{S_A}{S_L}. \qquad (2)$$

The efficiency is the ratio of the actual mechanical advantage (AMA) to the ideal mechanical advantage (IMA).

$$\% \text{ efficiency} = \eta = \frac{\text{AMA}}{\text{IMA}} \times 100 = \frac{\text{work done at load}}{\text{work done by effort}}. \qquad (3)$$

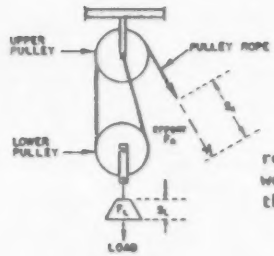


Fig. 2

Consider the two-pulley block systems shown in Figure 2.

If we wish to lift the load one foot, we must shorten the rope on each side of the lower pulley by one foot. To do this, we must move the applied force a total of two feet. Therefore, the distance (S) relationship is:

$$S_A = 2 S_L$$

and, because the time of movement is the same for the load and the effort, the velocity (V) relationship becomes:

$$V_A = 2\,V_L$$

If we ignore any losses in the pulleys, the input (applied) and output (load) work (W) must be equal:

$$W_A = W_L$$

$$F_A S_A = F_L S_L$$

But because $S_A = 2\,S_L$, we may write:

$$2\,F_A \cancel{S_L} = F_L \cancel{S_L}$$

or

$$F_A = \frac{1}{2} F_L$$

In other words, the applied force F_A is only half of the load force F_L. Comparing this result to the simple pulley, we see that the pulley block not only changes direction, but also decreases the force required to move the load.

COMPOUND PULLEY SYSTEMS

It is possible to build a pulley block with any reasonable number of pulleys. Figure 3 shows several possibilities.

It is possible to work out a general relationship for distances, velocities and forces. To do this, we observe that, in each case, the number of pulley ropes that must be shortened to move the load upward is always equal to the number of pulleys (N). Therefore, we have:

$$S_A = NS_L \qquad (4)$$

where N is the number of pulleys in the block. Also:

$$V_A = NV_L \qquad (5)$$

and

Fig. 3

$$F_A = \frac{1}{N} F_L \qquad (6)$$

The force relationship only holds for frictionless pulleys.

Actually, in most pulley blocks the pulley friction cannot be completely ignored. It usually can be accounted for most readily by measuring the effort (F_A) and the load (F_L). For example, suppose that we had a six-pulley block and we wished lift a 540-lb load. According to Equation 6, the effort should be:

$$F_A = \frac{1}{N} F_L$$

$$= \frac{1}{6} 540$$

$$F_A = 90 \text{ lb}$$

Thus only 90 lb are required to move the load.

Suppose however, that it actually takes 100 lb to raise the load. Knowing this we can calculate the efficiency (η) of the block.

$$\eta = \frac{\text{work done at load}}{\text{work done at effort}}$$

$$= \frac{F_L S_L}{F_A S_A}$$

$$\eta = \frac{F_L}{F_A} \times \frac{S_L}{S_A} \qquad (7)$$

However, we see from Equation 4 that:

$$S_A = NS_L$$

and

$$\frac{S_L}{S_A} = \frac{1}{N}$$

Substituting S_L/S_A into Equation 7, the efficiency becomes:

$$\eta = \frac{F_L}{F_A} \times \frac{1}{N} \qquad (8)$$

or in our example

$$\eta = \frac{1}{6} \times \frac{540}{100} = 0.9$$

That is to say, the block in question is 90% efficient. In other words, 10 percent of the input or applied work is lost in pulley friction.

Another way to look at Equation 8 is to solve for the effort or applied force:

$$F_A = \frac{F_L}{N\eta} \qquad (9)$$

This is a more practical relationship for effort in terms of the load, number of pulleys and efficiency.

MATERIALS

Assortment of pulleys that can be connected in different combinations.
Weight and weight pans.
Spring balance.

PROCEDURE

1. Construct a two-pulley block as shown in Figure 4.
2. Place a fixed weight (weights #1) on weight pan (another spring balance may be used in place of weights).
3. With a spring balance measure force required to lift load with pulley. (Load should be lifted slowly and smoothly, without sudden changes in speed, or force reading will be incorrect!!)
4. Complete data table for fixed weights #1 and two-pulley block.
5. Repeat steps 1 through 4 for a three- and four-pulley block system, still using fixed weights #1.
6. Repeat steps 1 through 5 for another load F_L, weights #2.

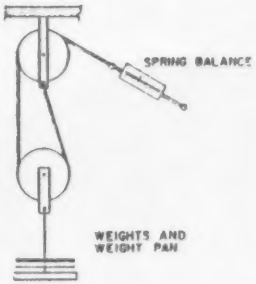


Fig. 4

	DATA TABLE														
	Two-Pulley Block					Three-Pulley Block					Four-Pulley Block				
Test	F_L	F_A	$\frac{F_L}{F_A}$	N	η	F_L	F_A	$\frac{F_L}{F_A}$	N	η	F_L	F_A	$\frac{F_L}{F_A}$	N	η
#1															
#2															

SELF CHECK

1. A six-pulley block system lifts a 95-lb load. The effort (applied force) required is 40 lb. What is the efficiency of the pulley block?
2. The force applied to a three-pulley block is 240 lb. If the pulley block is 80 percent efficient, what is the load?
3. We wish to lift a 790-lb load using a 100-lb applied force. What is the minimum number of pulleys that can be used if the efficiency is to be no less than 80 percent?

REFERENCES

Tinnell, Richard W. Mechanism-Drives. Albany, NY: Delmar Publishers, 1971.

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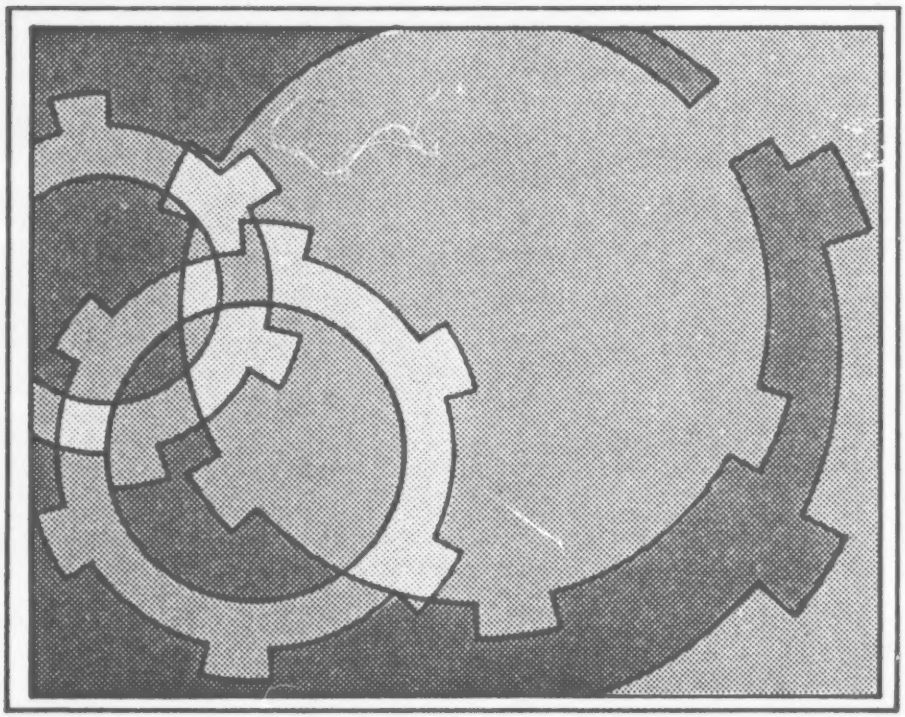
UNIFIED TECHNICAL CONCEPTS

MODULE 8 M 6

DIFFERENTIAL HOIST



MECHANICAL APPLICATION OF

FORCE TRANSFORMERS

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

In Module 8M5, the mechanical principles of pulley systems, including the block and tackle, were investigated. It was demonstrated that large weights could be moved with reasonable forces by clever utilization of pulley systems.

By incorporating the wheel and axle as explained in Module 8-0, "Force Transformers," into pulley systems similar to those explained in Module 8M5, significant mechanical advantages can be obtained.

In this module, we shall investigate the differential hoist, or chain hoist as it is more commonly known.

PREQUISITES

Student should have completed Concept Modules 8-0, "Force transformers," and Application Module 8M5, "Block and Tackle."

OBJECTIVES

To complete this module, the student should be able to:

- Describe (with the aid of suitable diagrams) the differential hoist; explain how it acts as a force transformer.
- Given a differential hoist system, calculate its actual mechanical advantage and efficiency.

DISCUSSION

Probably the most familiar differential hoist is the chain hoist used in auto repair shops and in factories (Figure 1).

In a differential hoist of this type the differential pulleys at the top are composed of two parts: a large-diameter (D) pulley, and a small-diameter (d) pulley. The two pulleys are fixed so that they turn together. A chain normally is used in this type of differential hoist, and the pulleys are notched so that the chain cannot slip.

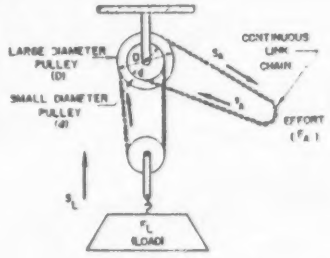


Fig. 1

To raise the load, the side of the chain that is looped over the large upper pulley is pulled. This causes both upper pulleys to turn through a common angle θ. In doing this the applied force moves through a distance S_A. This distance is related to the upper pulley rotation by:

$$S_A = R\theta = 1/2\ D\theta \qquad (1)$$

At the same time the other side of the chain passes over the smaller upper pulley. And, as this pulley also turns through the angle θ, the lower half of the chain loop moves a distance s_A given by:

$$s_A = r\theta = 1/2\ d\theta \qquad (2)$$

Both of these movements are transmitted to the load. It tends to move up a distance S_A and down a distance s_A simultaneously. The total load movement (S_L) is half the difference in these two values because the total difference must be divided equally between the two supporting lines.

$$S_L = 1/2\ (S_A - s_A) \qquad (3)$$

With these three equations, the ratio of S_A to S_L (the ideal mechanical advantage) can be determined. Dividing Equation 1 by Equation 3 yields:

$$IMA = \frac{S_A}{S_L} = \frac{\cancel{1/2}\ D\theta}{\cancel{1/2}\ (S_A - s_A)}$$

and substituting Equations 1 and 2 for S_A and s_A respectively on the right side gives:

$$IMA = \frac{S_A}{S_L} = \frac{\cancel{D\theta}}{1/2\ \cancel{D\theta} - 1/2\ d\theta}$$

And if we factor and cancel the common terms, we obtain:

$$IMA = \frac{S_A}{S_L} = \frac{2D}{D - d} \qquad (4)$$

In Equation 4 one should notice that, if diameters are nearly equal, the ratio of S_A to S_L becomes very large.

Differential chain hoists of this type have low efficiencies of 30 to 40 percent due to frictional losses but are capable of lifting large loads easily. We can calculate the efficiency (η) by recalling that:

$$\eta = \frac{\text{work done on load}}{\text{work done by applied force}}$$

And, as work is force times distance in each case,

$$\eta = \frac{F_L S_L}{F_A S_A}$$

or using Equation 4,

$$\eta = \frac{F_L}{F_A} \times \frac{(D - d)}{2D} \qquad (5)$$

Solving for F_A gives:

$$F_A = \frac{F_L}{\eta}\left(\frac{D - d}{2D}\right)$$

The hand-operated chain hoist is used very widely for lifting loads up to about 1000 lb. If diameters of the top pulleys are the same, the load will hold at any height to which it is raised; otherwise care must be taken to keep the applied force on the chain to prevent the load from lowering (Example A). This distance ratio can be advantageous when it is required to lower a load gradually to a precise position (such as an engine onto its bolt mounts) because small increments of lowering distance can be made accurately.

EXAMPLE A: DIFFERENTIAL HOIST CALCULATIONS

Given: The upper pulleys of a chain hoist are 8 inches and 9 inches in diameter respectively. The hoist is 80% efficient.

Find: The maximum weight of auto engine that can be lifted by an applied force of 50 lb. How much chain must be pulled through the pulleys to raise the load by 2 feet?

Solution: The ideal mechanical advantage (IMA) of this hoist is, from Equation 4:

$$IMA = \frac{S_A}{S_L} = \frac{2D}{D - d} = \frac{2 \times 9 \text{ in}}{(9 - 8) \text{ in}} = 18$$

As it is 80% efficient, the actual mechanical advantage (AMA) is:

$$AMA = \eta \times IMA = 0.80 \times 18 = 14.4.$$

Hence an applied force of 50 lb can lift an auto engine weighing:

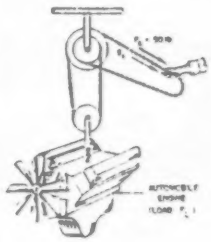
$$F_L = AMA \times F_A = 14.4 \times 50 \text{ lb} = 720 \text{ lb}$$

To find the distance through which the chain must be pulled, we recall from the principle of work that:

$$IMA = \frac{S_A}{S_L}$$

Here: IMA = 18 and S_L is 2 ft.

so: $S_A = IMA \times S_L = 18 \times 2 \text{ ft} = 36 \text{ ft}.$



MATERIALS

Differential chain hoist (an assortment of pulleys that can be used to construct a differential hoist).
Four different weights and appropriate weight pans.
Spring balance.

PROCEDURE

1. Set up differential hoist (chain hoist) as shown in Figure 2.
2. Place a fixed weight on weight pan (another spring balance may be used in place of weight, weight-pan combination). Identify weights plus weight pan as load #1.
3. With a spring balance, measure force required to lift load #1 with differential hoist.
4. Complete data table.
5. Repeat steps 1 through 4 for a second load weight, identified as load #2. Complete data table.

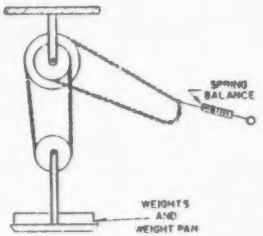


Fig. 2

DATA TABLE

D	d	IMA = 2D/D - d

	F_A	F_L	$IMA = \frac{S_A}{S_L} = \frac{2D}{D-d}$	$AMA = \frac{F_L}{F_A}$	$\eta = \frac{AMA}{IMA}$
Load #1					
Load #2					

SELF CHECK

1. Describe (with the aid of suitable diagrams) the differential hoist; explain how it acts as a force transformer.
2. A differential chain hoist has upper pulley diameters of 12 inches and 10 inches. What applied force is required to raise a 790-lb load if the efficiency is 35%?
3. An applied force of 175 lb is just able to support a load on a differential chain hoist. If the effective pulley diameters are 12 inches and 10 inches while the efficiency is 33%, what is the size of the load?
4. A chain hoist that has upper pulleys of diameters 25 cm and 20 cm and an efficiency of 75% is used to raise a 100-kg load through 2 meters.
   a. How much chain must be pulled through the system?
   b. What is the minimum applied force that is required?

REFERENCES

Tinnell, Richard W. Mechanism-Drives. Albany, NY: Delmar Publishers, 1971.

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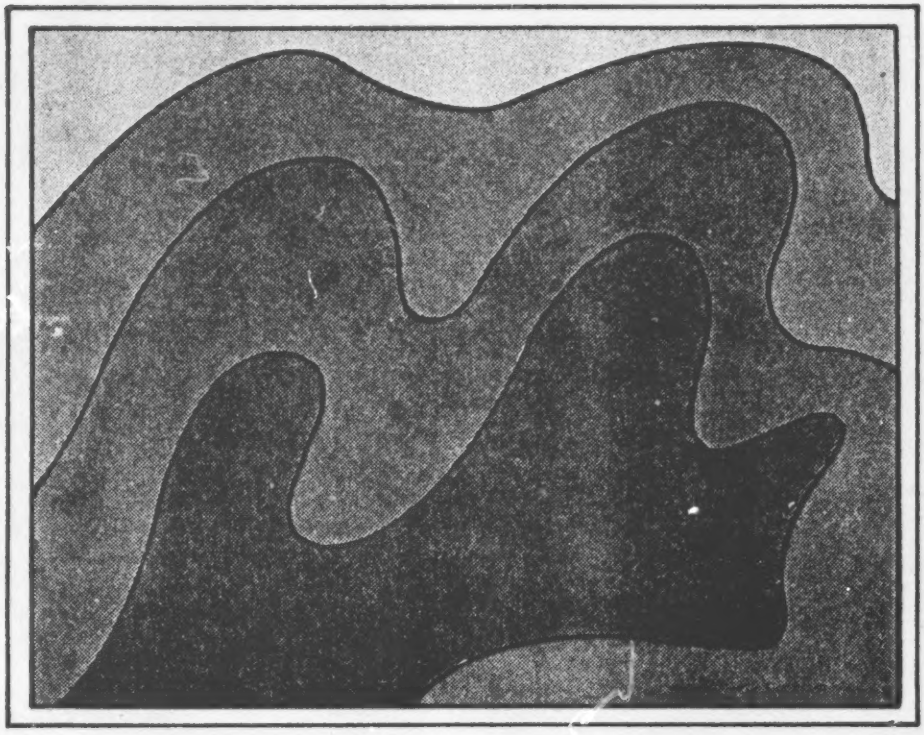
UNIFIED TECHNICAL CONCEPTS

MODULE 8 F 1

HYDRAULIC PRESS



FLUID APPLICATION OF FORCE TRANSFORMERS

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

In one form or another, force transformers are used every day. They may not be identified as such, but lifting an automobile in the gas station repair area, pressing the brake pedal, or stamping out sheet metal parts in a factory, are all examples of simple force transformers in the fluid system. They all utilize the same physical principle — the hydraulic press — to gain a force advantage when the applied force must move through a greater distance than the load. This module introduces the hydraulic press. In the laboratory, the student will construct and operate a hydraulic-press simulator.

PREREQUISITES

Student should have completed Concept Module 8-0, "Force Transformers."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:

  Closed hydraulic system.
  Hydraulic force transformation.
  Mechanical advantage.
  Hydraulic advantage.

- Using illustrations, describe Pascal's law.

- Given diameter of two pistons in a closed hydraulic system, calculate:

  Distance moved by one, given distance moved by the other.
  Lift of one, given the force applied to the other.

- List at least two examples of mechanisms that apply Pascal's law.

- Given the appropriate equipment, demonstrate the characteristics of a hydraulic press.

CLOSED HYDRAULIC SYSTEMS

A closed hydraulic system contains a rigid tank similar to the one in Figure 1, completely filled with a liquid and vented to several pistons as shown. There are no leaks of liquid through the piston sides, no air space at the top of the tank, and no liquid pumped in or out during operation.

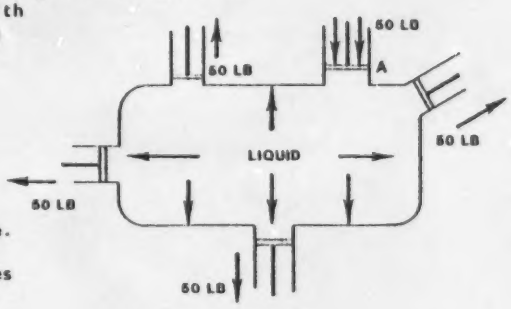


FIG. 1

If any of the pistons are moved in or out or up or down, one or more of the other pistons would have to move. This is because the liquid is not compressible. (Note that in an enclosed pneumatic system — air, not liquid — the air compresses easily, and the same laws do not apply.)

If piston A is moved inward, one piston, or a combination of the other pistons, would have to move outward, as no liquid or air could seep in, and the total volume of the system would remain the same.

PASCAL'S PRINCIPLE

According to Pascal's law, the pressure in an enclosed liquid tank is essentially the same everywhere within the tank. Furthermore, pressure is the same in all directions at any point; for if the pressure were different, there would be a flow of liquid until the pressure was equalized.

As a consequence of Pascal's law, two affects in the tank in Figure 1 can be observed. When an inward force of 50 lb is applied at piston A, it produces a pressure that is transmitted equally throughout the tank. A pressure measurement made anywhere in any direction would be the same. Furthermore, if all the pistons shown had the same area, the outward force of each would be 50 lb, the same as the force applied inwardly on piston A.

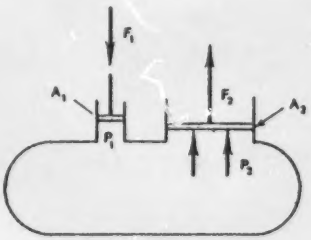


FIG. 2

FORCE MULTIPLICATION

Consider a tank with two pistons of different diameters, as a means of multiplying force.

In Figure 2, the force applied at piston A will produce a

pressure in the tank. By Pascal's law, this pressure is transmitted equally throughout the fluid and, hence, is the same at piston A_2.

The basic formula for pressure is given by Equation 1.

$$p = \frac{F}{A} \tag{1}$$

where

p = pressure in pounds per square inch.
F = force applied in pounds.
A = area to which the force is applied in square inches.

Because pressure is the same everywhere in a closed hydraulic system, the pressure at piston 1 is the same as the pressure at piston 2, as indicated by Equation 2.

$$p_1 = p_2 \tag{2}$$

where

p_1 = pressure at piston 1.
p_2 = pressure at piston 2.

The pressure at piston one is shown by Equation 3.

$$p_1 = \frac{F_1}{A_1} \tag{3}$$

where

F_1 = force on piston 1.
A_1 = area of piston 1.

Similarly, Equation 4 gives the pressure at piston 2.

$$p_2 = \frac{F_2}{A_2} \tag{4}$$

where

F_2 = force at piston 2.
A_2 = area of piston 2.

Since $p_1 = p_2$, the force at piston 2 can be calculated by Equation 5.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad (5)$$

or

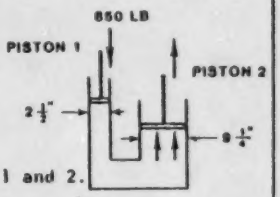
$$F_2 = \left(\frac{A_2}{A_1}\right) F_1$$

Thus the force at piston two can be greater than the applied force at piston one if $A_2 > A_1$. The ratio A_2/A_1 is called the mechanical advantage of the system. An example of the use of these equations is given in Example A.

EXAMPLE A: CALCULATION OF A FORCE MULTIPLICATION PROBLEM

Given: An 850-lb force is applied to piston 1, which has a diameter of 2.5". Piston 2 is 9.25" in diameter.

Find: Force applied on piston 2.



Solution: Using Equation 5:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Calculate the area of pistons 1 and 2.

Piston 1 area $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} (2.5)^2$ $\quad \pi = 3.14$, $d = 2.5$ inches

$= \left(\frac{3.14}{4}\right)(6.25)$

$A_1 = 4.91$ square inches

Similarly:

Piston 2 area $= \frac{\pi}{4} (9.25)^2$

$= \left(\frac{3.14}{4}\right)(85.5)$

$A_2 = 67.2$ square inches

Rearranging Equation 5 gives:

$$F_2 = \left(\frac{A_2}{A_1}\right) F_1 = \left(\frac{67.2}{4.91}\right)(850)$$

$F = 11{,}636$ lb

Therefore, an 850-lb force on piston 1 will lift a force of 11,636 lb on piston 2.

EFFECT OF FORCE TRANSFORMATION ON DISTANCE

The analysis of work involved shows that work in equals work out. Work is equal to force applied times distance moved. If more force is put out, the distance traveled by the force will be less; therefore, except for frictional losses, the work output equals the work input. Force, not energy, is multiplied.

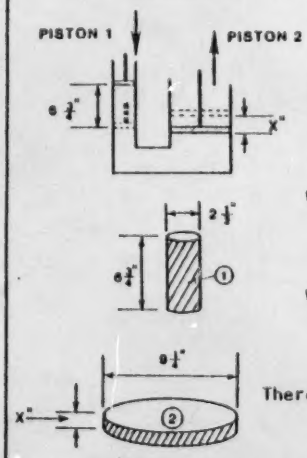
Consider again Example A, from the standpoint of distance traveled by each of the two pistons. See Example B.

EXAMPLE B: CALCULATION OF DISTANCE FORCES TRAVEL USING EXAMPLE A

Given: Same hydraulic system as in Example A, and piston 1 moves 6 3/4".

Find: How far piston 2 moves.

Solution: (a) The solution to this problem may be found by assuming that the total volume of the system remains the same during piston movement; therefore —



Volume of liquid displaced by the movement of piston 1 = volume of liquid displaced by the movement of piston 2

Volume of piston 1 = $(\frac{\pi}{4} d_1^2)(6\ 3/4)$

= (4.9 sq in)(6.75)

V = 33 cubic inches

Volume of piston 2 = $\frac{\pi}{4} d_2^2(x)$

V = 67.2 (x)

Volume 1 = Volume 2

Therefore,

$$33 = 67.2\ (x)$$

$$x = \frac{33}{67.2} = 0.5 \text{ in}$$

Note that in raising such a large weight of 11,636 lb, piston 2 only moves 1/2".

(b) A somewhat similar approach uses the work principle, as follows:

$$F_1 x_1 = F_2 x_2$$

$$x_2 = (\frac{F_1}{F_2})\ x_1$$

$$= \left(\frac{850}{11,636}\right)(6.75)$$

$$x_2 = 0.5 \text{ in}$$

A hydraulic press is a further development of the hydraulic jack principle (Figure 3). If a frame is bolted to the hydraulic jack, considerable pressure can be exerted between the two platens to bend of squeeze metal into a variety of desired shapes.

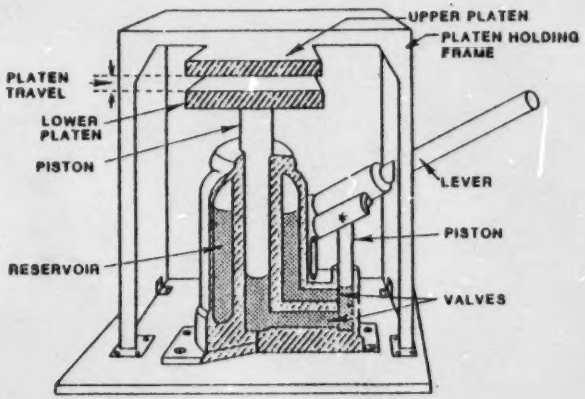


FIG. 3

SUMMARY

The hydraulic press is a force transformer that transforms an applied force into a greater force at the output. The applied force must pass through a greater distance than the load. The force upon the liquid in a hydraulic press is the same in all directions for every square inch; however, when an output piston is larger (in cross-sectional area) than an input piston, the output piston exerts greater force, because more area is exposed to liquid pressure.

SIMULATION OF HYDRAULIC PRESS

MATERIALS

60-cm^3 disposable syringe — eccentric tip without needle.
12-cm^3 disposable syringe — eccentric tip without needle.
Eight 500-gm slotted weights.
Two each 200-g, 100-g and 50-g slotted weights.
One 5-gallon bucket or plastic container.
Flexible rubber or plastic tubing, clear 1/16 inch — inside diameter, 1/32 inch wall thickness (medical supply house).
Camps to secure hose to syringes.
Two ring stands and two clamps to support syringes in vertical position.
Vernier caliper.

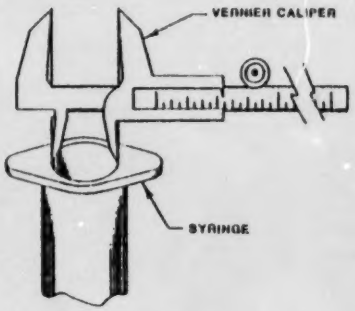


FIG. 4

PROCEDURE

Construct a simulated hydraulic press as follows:

1. Remove plungers from both 12-cm^3 and 60-cm^3 syringes. Measure inside diameter of cylindrical barrels with vernier calipers (Figure 4). Record these values in Data Table.

2. Place the two unassembled syringe barrels and plungers and a twelve-inch length of tubing into 5-gallon container full of water.

3. Assembly of hydraulic press simulator in this experiment is made totally under water, so that no air is allowed to leak into closed systems. With 60-cm^3 syringe under water, push its plunger all the way to the botton to exhaust any air; fill with about 10 cm^3 of water. Leave it submerged at bottom of bucket; repeat with the 12-cm^3 plunger. Connect tubing, filled with water, to eccentric tips of both plungers, while holding both syringes

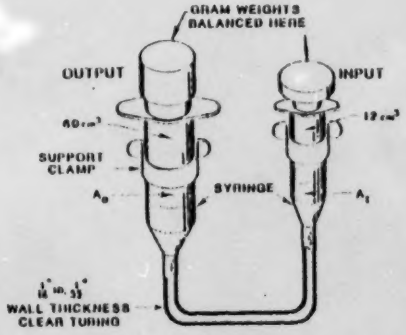


FIG. 5

and tubing under water. Remove apparatus from water; mount syringes in a vertical position, making sure that they are secure and well supported (Figure 5).

4. Check operation of hydraulic system by pushing down manually first on one plunger, then on the other. If no air is in hydraulic system, plungers should move immediately and simultaneously.

5. System assembled (Figure 5) may be termed a hydraulic press simulator. The 60-cm^3 syringe represents output of simulator; and 12-cm^3 syringe represents input. Manually pushing input plunger requires a certain amount of force to make output plunger move. Force required is that necessary to overcome frictional resistance of hydraulic system; that is, drag of rubber plungers. To measure this breakway force, proceed as follows. With no weights on output plunger, carefully balance slotted weights on input plunger until weight is just sufficient to make both plungers begin to move and continue to move slowly. Record this weight (breakaway force) in Data Table. Remove weights; return fluid to input syringe (approximately 6-8 cm^3).

6. Carefully balance three 500-g slotted masses on output plunger, while manually preventing input plunger from being forced out of its tube. Place frictional force weight measured in No. 5 on input syringe; carefully stack just enough additional slotted weights on top of input plunger to start system moving slowly. This procedure should show clearly that a given force on input side causes a greater force (output weight) to be overcome. Record total force on input plunger in Data Table. Note: Under loaded condition of step 6, if weights are not carefully balanced and syringe barrels kept vertical, plungers rub against barrels, causing additional friction and adding to instability of apparatus. Error thus may be introduced into measurements associated with this experiment.

7. Repeat process outlined in steps 5 and 6 for two additional output weights; record all data in Data Table. Note: In this experiment masses are loaded on plungers. The fact that the following ratios are equal allows use of masses and weights interchangeably:

$$\frac{\text{Output mass load}}{\text{Input mass load}} = \frac{\text{Output force}}{\text{Input force}}$$

8. Complete the following exercises:

   (a) Calculate cross-sectional area of the output and input syringes with the following equation:

$$A = \frac{\pi}{4} d^2$$

Record results in Data Table.

(b) For each trial in the experiment, calculate difference between the input mass for that trial and the input mass from trial 1 (no output load). Record these differences in Data Table.

(c) Calculate ratio of the output force (mass) to the output area; record in Data Table.

(d) Calculate ratio of the true input force (i.e., total weight less frictional force) to the output force; record in the Data Table.

(e) Calculate ratio of the input cross-sectional area to the output cross-sectional area; compare the ratios obtained in exercises d and e. Record in the Data Table (see Equation 2).

DATA TABLE

Diameter of 60-cm^3 output syringe = _____ cm

Diameter of 12-cm^3 input syringe = _____ cm

Cross-sectional area of output (A_o) = _____ cm^2

Cross-sectional area of input (A_i) = _____ cm^2

Trial	Mass Unloaded On Output (g)	Mass Loaded On Input (g)	Input Mass Minus Frictional Mass
1	0		0
2			
3			
4			

F_o/A_o = _____ dynes/cm^2

F_i (true)/F_o = _____

A_i/A_o = _____

How closely does $\frac{F_i}{F_o} = \frac{A_o}{A_i}$?

SELF CHECK

1. Write a short description of the following terms, include any sketches to clarify presentation:

   Closed hydraulic system.
   Hydraulic force transformation.
   Mechanical advantage.
   Hydraulic pressure.

2. Define Pascal's law, using illustrations to describe the principles involved.

3. If a load of 1000 lb is moved through 21 inches on the input piston, how much load will it lift on the output piston if the input and output piston diameters are 1 inch and 12 inches respectively? How far will the 12-inch piston (output piston) move during this process?

4. Give two everyday examples of the use of Pascal's law.

REFERENCES

Beiser, Arthur. Modern Technical Physics. 3rd ed. Menlo Park, CA: The Benjamin/Cummings Publishing Company, Inc., 1979.

McNickle, L.S., Jr. Simplified Hydraulics. New York: McGraw-Hill Book Company, 1966.

Mobile Hydraulics Manual, M-2990. Troy, MI: Sperry Vickers Corporation.

Stewart, H.L. Practical Guide to Fluid Power. Indianapolis, IN: Theodore Audel and Co., Division H. W. Sams, 1968.

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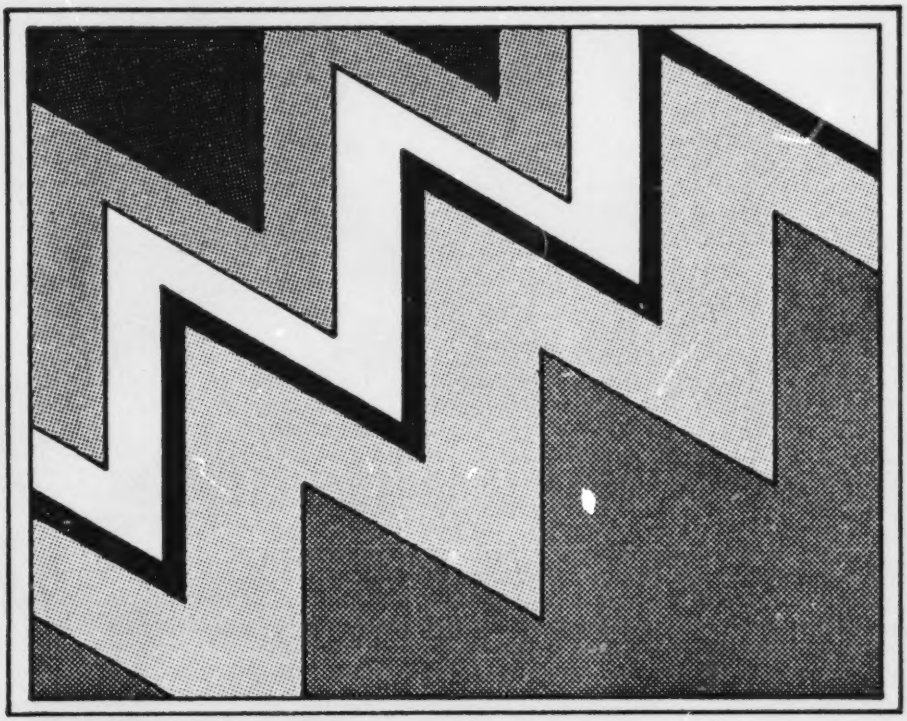
UNIFIED TECHNICAL CONCEPTS

MODULE 8 E 1

ELECTRICAL TRANSFORMERS



ELECTRICAL APPLICATION OF FORCE TRANSFORMERS

Technical Education Research Center – Southwest

TERC – SW

INTRODUCTION

The transformer, whose principal function is to increase or to decrease the voltage available from a source of alternating current, was one of the earliest electrical components, and is still one of the most widely-used ones.

This module introduces the operating principles of the transformer. Voltage, current, power relationships, and transformer power losses also will be investigated. In the laboratory, the student will construct an electrical transformer and will determine its characteristics.

PREREQUISITES

Student should have completed Concept Module 8-0, "Force Transformers," and Application Module 3E1, "Electric Current."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given the turns ratio of the coils in a transformer, current (I_p) and voltage (V_p) input, calculate current (I_s) and voltage (V_s) output of the secondary coil.
- Given either voltages or currents in the primary and secondary coils of a transformer, calculate ratio of turns of the coils.
- Briefly discuss the three major sources of power loss in a transformer.
- Given the secondary resistance (R_s) of a transformer and the turns ratio of the coils, calculate the primary resistance (R_p).
- Given the appropriate equipment, determine the operating characteristics of an electrical transformer.

DISCUSSION

The principal function of a transformer is to increase or to decrease the voltage available from a course of alternating current, accomplished through magnetic induction. The changing of voltage in one wire loop will induce a voltage in a nearby loop. This induced voltage is due to the changing magnetic field that joins the two loops.

A simple transformer, as shown in Figure 1, consists of three principal parts: one, a primary coil connected to a source of alternating current; two, a secondary coil attached to a load (light bulb, appliance, etc.); and, three, a soft iron core.

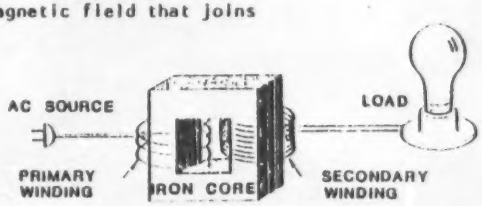


FIG. 1

The a.c. voltage source of the primary winding generates an alternating magnetic flux in the core that induces an a.c. voltage in the secondary winding. The magnitude of voltage in the secondary is dependent upon the number of turns in the two coils. Schematically, this simple transformer could be drawn as shown in Figure 2.

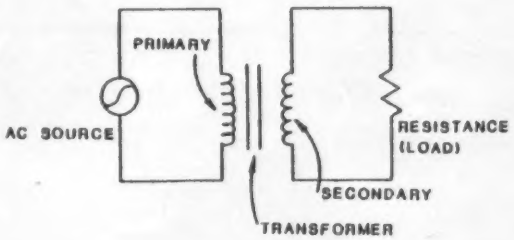


FIG. 2

CLASS OF TRANSFORMERS

Depending on whether they increase or decrease the voltage applied to them, there are essentially two classes of transformers — one that increases voltage, called a "step-up transformer," and one that decreases voltage, called a "step-down transformer." How a transformer increases or decreases voltage is explained as follows: Assuming that the same number of magnetic flux lines pass through both coils, the volts per turn in each coil would be the same. Therefore, the ratio of the secondary voltage to the primary voltage is equal to the ratio of the number of turns in the secondary coil to the number of turns in the primary coil. Equation 1 shows this transformer law.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \qquad (1)$$

where

V_s = secondary voltage.

V_p = primary voltage.

N_s = number of turns in secondary coil.

N_p = number of turns in primary coil.

Thus, a step-down transformer has fewer turns in the secondary coil than in the primary coil, and a step-up transformer has more. Typical transformer problems are solved in Examples A and B by use of Equation 1.

EXAMPLE A: STEP-UP TRANSFORMER VOLTAGE

Given: The transformer in Figure 1 has 4 turns in the primary winding and 8 turns in the secondary winding. The input voltage to the primary coil (V_p) is 3 volts.

Find: Theoretical output voltage from the secondary coil (V_s).

Solution:

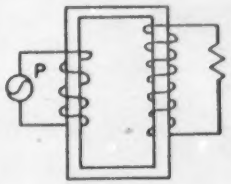
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_s = \left(\frac{N_s}{N_p}\right) V_p$$

$$= \left(\frac{8 \text{ turns}}{4 \text{ turns}}\right) 3 \text{ V}$$

$$= (2)(3V)$$

$$V_s = 6V$$



The transformer "steps up" the voltage from 3 volts to 6 volts.

EXAMPLE B: STEP-UP TRANSFORMER WINDING

Given: An electric generator delivers energy at a voltage of 15,000 volts.

Find: The additional number of turns needed in the secondary winding than in the primary winding to step this voltage up to 300,000 V. (Assume 100% efficiency.)

Solution: $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{300,000}{15,000} = 20$

$N_s = (20)(N_p)$

The secondary winding will need 20 times as many turns as the primary winding.

As long as the voltage rating of the winding insulation is not exceeded, either winding of a transformer can be used as the primary winding. A step-up transformer can be used as a step-down transformer if its leads are reversed. For example, if the transformer in Example A is reversed by putting 3 V into the side with eight turns, the voltage out of the four-turn winding side will be 1.5 V.

POWER

At first glance, it would seem possible to raise electrical energy to enormous levels through sequential use of step-up transformers. This would be a violation of the conservation-of-energy principle. A transformer does not create energy, it merely transforms it. The rate of transfer is called "power," defined by Equation 2.

$$P = IV \qquad (2)$$

where

P = power.
I = current.
V = voltage.

If a transformer increases voltage, it decreases current proportionately, and vice versa; therefore, power is conserved and can be written in a number of useful ways:

Power in = power out

$$I_p V_p = I_s V_s$$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

If this last equation is considered in light of Equation 1, the transformer law can be written in terms of current, as shown in Equation 3.

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \qquad (3)$$

where

I_s = current through secondary coil.

I_p = current through primary coil.

The ratio of turns in Equation 3 is inverse to the ratio in Equation 1. The use of Equation 3 for solving problems is shown in Example C.

EXAMPLE C: STEP-UP TRANSFORMER CURRENT

Given: A transformer has 500 primary turns (N_p) and 3000 secondary turns (N_s).

Find: The secondary current (I_s) if the primary current (I_p) is 3 A.

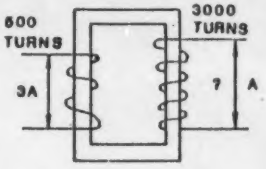
Solution:

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$I_s = \left(\frac{N_p}{N_s}\right)$$

$$I_p = \left(\frac{500}{3000}\right) 3 \text{ A}$$

$$I_s = 0.5 \text{ A}$$



3 A is reduced to 0.5 A in a step-up transformer.

To summarize: if the voltage is stepped up, the current is stepped down; if the voltage is stepped down, the current is stepped up.

USE OF OHM'S LAW

Ohm's law, however, states that an increase in voltage corresponds to an increase in current. Is this a contradiction? No, Ohm's law still applies, as the voltages are fixed values. Primary voltage is equal to the voltage of the a.c. source, and the primary current is determined by what happens in the secondary circuit. Secondary voltage is determined by the ratio of the loops in the primary and secondary coils. The ratio is a fixed value; and the secondary current obeys Ohm's law, equal to the induced voltage divided by the load (resistance). The product of the induced voltage and the secondary current is equal to the power in the secondary coil. Since in the ideal case, power in = power out, the primary current then can be determined. Example D explains this further.

EXAMPLE D: OHM'S LAW IN A TRANSFORMER

Given: A transformer has twice as many secondary loops (N_s) as primary loops (N_p). The primary coil is powered by a 100 V (V_p) source and the secondary coil is connected to a 50 Ω load.

Find: (a) Induced voltage (V_s) in the secondary. (b) Current (I_s) in the secondary. (c) Power (P_s) in the secondary. (d) Current (I_p) in the primary. (e) Is this a step-up or step-down transformer.

Solution: (from Equation 1)

(a) $$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_s = \left(\frac{N_s}{N_p}\right) V_p$$

$$V_s = (2)(100\ V) = 200\ V$$

(b) $V_s = IR$ (Ohm's law)

$$I_s = \frac{V_s}{R_s} = \frac{200\ V}{50\ \Omega} = 4\ A$$

(c) $P_s = I_s V_s$

$$P_s = (4\ A)(200\ V) = 80 \text{ watts}$$

(d) Power in = power out (ideal transformer)

$$I_p V_p = I_s V_s$$

$$I_p = \left(\frac{V_s}{V_p}\right) I_s$$

$$I_p = \left(\frac{200\ V}{100\ V}\right) 4\ A$$

$$I_p = 8\ A$$

or

$$P_p = I_p V_p$$

$$I_p = \frac{P_p}{V_p}$$

$$= \frac{800\ W}{100\ V}$$

$$I_p = 8\ A$$

(e) Since $V_s > V_p$, this is a step-up transformer.

Note: Voltage is stepped up, current is stepped down, power is conserved, and Ohm's law is obeyed.

There are situations in which higher voltages are advantageous, and other situations in which low voltage is more useful. High voltage is necessary in power-transmission lines to keep heat losses to a minimum, and low voltage is necessary in most household appliances to keep them safe. Additionally, low voltage is necessary to yield the high currents needed for welding. These voltages are produced by the proper use of step-up and step-down transformers.

POWER TRANSMISSION

Consider the case of a power plant contracted to deliver 12,000 watts of electrical power (P) over a transmission line that has a resistance of 0.2 ohms (R). Consider further that the power plant unwisely decides to deliver the power at a potential of 120 volts (V). How effective will this power transmission be?

Remembering that P = IV, the current (I) in the transmission line is equal to the following:

$$I = \frac{P}{V} = \frac{12{,}000\ W}{120\ V} = 100\ A$$

From Ohm's law, $V = IR$, the voltage drop in the transmission line is —

$$V = IR = (100\ A)(0.2\ \Omega) = 20\ V$$

This calculation means that the customer gets the power, not at 120 V, but at 120 V - 20 V or 100 V.

Power loss due to heat in the transmission line varies as the square of the current (I). At 100 A, the power loss due to heat is given thusly:

$$P = I^2R = (100\ A)^2\ (0.2\ \Omega) = 2000\ W$$

The customer only gets 12,000 W - 2,000 W, or 10,000 W, instead of the contracted amount.

Because a transformer was not utilized to step-up the transmission voltage, the power dropped from 12,000 W to 10,000 W, and the voltage dropped from 120 V to 100 V. Both are substantial losses. Example E shows how much more efficient the power transmission can be made by use of a step-up transformer and why transformers are extremely valuable in power transmission.

EXAMPLE E: POWER TRANSMISSION AT HIGH VOLTAGE

Given: A power plant contracted to deliver 12,000 watts of electrical power (P) over a transmission line that has a resistance of 0.2 ohms (R). Through the proper use of a step-up transformer, the potential can be raised from 120 volts to 12,000 volts (V).

Find: (a) Current (I) in the transmission line. (b) Voltage drop along the line. (c) Power loss due to heat. (d) Power and voltage the customer ultimately receives. (Assume that the voltage is stepped back down proportionately at the customer's place of business.)

Solution: (a) $P = IV$

$$I = \frac{P}{V} = \frac{12{,}000\ W}{12{,}000\ V}$$

$$I = 1\ A$$

(b) $V = IR$

$= (1\ A)(0.2\ \Omega)$

$V = 0.2\ V$

(c) $P = I^2R$

$= (1\ A)^2\ (0.2\ \Omega)$

$P = 0.2\ W$

(d) Power to customer: 12,000 W - 0.2 W = 11,999.8 W

Voltage to customer:
1) Transmitted — 12,000 V - 0.2 V = 11,999.8 V
2) Stepped down — 11,999.8 V ÷ 100 = 119.998 V

The power and voltage drops an insignificant amount.

Ohm's law can be used one step further to develop a resistance equation for transformers. From Equations 1 and 3, expressions for primary voltage and current are derived as follows:

$$V_p = \left(\frac{N_p}{N_s}\right) V_s$$

and

$$I_p = \left(\frac{N_s}{N_p}\right) I_s$$

Since Ohm's law yields $R = V/I$ —

$$R_p = \frac{V_p}{I_p} = \frac{(N_p/N_s)\ V_s}{(N_s/N_p)\ I_s} = \left(\frac{N_p^2}{N_s^2}\right) R_s$$

Equation 4 shows the relationship between resistance and turns of both coils.

$$\frac{R_p}{R_s} = \left(\frac{N_p}{N_s}\right)^2 \qquad (4)$$

Example F demonstrates the use of this equation.

EXAMPLE F: RESISTANCE OF A TRANSFORMER

Given: A transformer has a secondary resistance (R_s) of 25 ohms, 400 turns in the primary winding, and 100 turns in the secondary winding.

Find: Resistance of the primary winding.

Solution:

$$\frac{R_p}{R_s} = \left(\frac{N_p}{N_s}\right)^2$$

$$R_p = \left(\frac{N_p}{N_s}\right)^2 R_s$$

$$= \left(\frac{400}{100}\right)^2 (25\ \Omega)$$

$$= (4)^2 (25\ \Omega)$$

$$R_p = 400$$

Note: The load resistance of 25 Ω does not include the d.c. resistance of the winding wire which is usually low compared to the load resistance. The load resistance mentioned here is usually called "impedance."

POWER LOSSES

In reality, not all the power put into a transformer is available at the secondary terminals. Some of the power is lost as heat in the transformer windings (copper losses), and some power is lost in the transformer core itself (core losses). Consequently, there are three major sources of power loss in a transformer.

I^2R LOSS

Coils of wire have resistance. When current flows through a resistance, power is lost due to heat. This power loss (P), as shown in Example E (c), can be calculated by the equation $P = I^2R$. Thus, power loss due to the winding resistance is called the "I^2R loss."

EDDY-CURRENT LOSS

Current in the primary winding induces current in the core as well as in the secondary winding. This core current, called "eddy current," dissipates power as it flows through the core material. Eddy-current loss can be reduced when the sheets of the core are laminated. If the core is made of thin sheets of magnetic material separated by insulating varnish, the length of the eddy current paths is reduced, thereby reducing the power loss. In some transformers, to reduce eddy current, the core is made of magnetic powder suspended in a ceramic-like, insulating material.

HYSTERESIS LOSS

Energy is required to reverse the direction of core magnetization every half cycle of the input frequency. This energy, called "hysteresis," is lost to heat like I^2R and eddy-current losses, and is controlled by the use of iron cores having low hysteresis susceptibility. Hysteresis losses are greater at higher frequencies, since the core magnetization must reverse more frequently. In an effort to reduce hysteresis loss at very high frequencies, an iron core is not used. The secondary and primary coils are simply wound on a nonmagnetic, nonconducting core.

Losses like these in a well-designed transformer do not normally exceed two or three percent; however, at these losses the power out of a transformer will always be less than the power in. The ratio of power out to power in is called the "efficiency." In all of the discussion and example problems to this point, an ideal efficiency of 100 percent has been assumed (power out = power in). For the sake of accuracy, the true transformer efficiency should be included.

In Figure 1, a transformer having windings on opposite sides of the transformer core was shown. In practice, the core is shaped with a middle core piece, as shown in Figure 3, with the secondary coil wound on top of the primary.

Transformers can be wound with many secondary windings or with several connection taps on one secondary winding to suit the application.

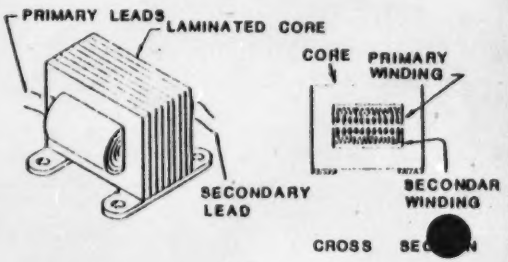


FIG. 3

SUMMARY

The electrical transformer steps up or steps down voltage through the process of magnetic induction. The a.c. transformer consists of (1) a coil connected to an a.c. source, (2) another (secondary) coil attached to a load, and (3) a soft iron core. The number of wire turns in each coil determines the amount of voltage change; the coil with the larger number of turns is capable of stepping up the voltage, and vice versa.

MATERIALS

Laminated iron transformer core.
Spool of conducting wire.
Voltmeter of volt-ohm-milliammeter (VOM) (Triplett 630 or Simpson 260).
Low voltage (0-12 V) a.c. power supply.
1 kΩ resistor.
Assorted test leads.

PROCEDURE

1. Form a primary coil for a transformer by winding 24 turns (N_p) of conducting wire around one part of a laminated iron core.

2. Form a secondary coil around the same core, but on opposite side, by winding 6 turns (N_s) of conducting wire.

3. Connect transformer to a circuit, as shown in Figure 4.

4. Plug in a.c. power supply, and adjust it to deliver 6 volts (V_p) to primary coil. Record the voltmeter reading (V_s) in the Data Table. Compare this experimental value of V_s with the value calculated using the data given for V_p, N_p and N_s. Repeat with V_p = 10 volts.

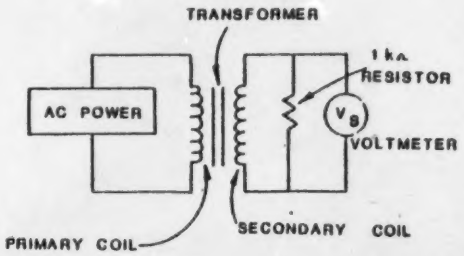


FIG. 4

5. Unplug the power supply. Rewind the secondary coil of the transformer with 12 turns of conducting wire. Repeat steps 3 and 4.

6. Increase the number of turns in the secondary coil to 36 and 48 turns. Record all data in the Data Table.

7. Using calculated values of secondary voltage (V_s), calculate secondary current (I_s), secondary power (P_s), and primary current (I_p). Assume the transformer to be 100% efficient (power in = power out).

8. Why is an a.c. current (or at least a pulsating current) necessary for transformer operation?

DATA TABLE								
Trial No.	Primary Turns (N_p)	Secondary Turns (N_s)	Primary Voltage (V_p) Volts	Secondary Voltage (V_s) (Calc) Volts	Secondary Voltage (V_s) (Meas) Volts	Secondary Current (I_s) Amperes	Secondary Power (P_s) Watts	Primary Current (I_p) Amperes
1	24	6	6					
			10					
2	24	12	6					
			10					
3	24	36	6					
			10					
4	24	48	6					
			10					

SELF CHECK

1. A transformer has N_p = 400 turns, N_s = 900 turns, V_p = 8 V and I_p = 0.4 A. Calculate I_s and V_s.

2. A transformer has N_p = 1000 turns, N_s = 200 turns, I_p = 0.8 A, V_p = 10 V. Calculate I_s and V_s.

3. A transformer has V_p = 12 V and V_s = 3 V. Calculate the turns ratio.

4. A transformer has I_p = 2 A and I_s = 0.1 A. Calculate the turns ratio.

5. Briefly describe the three major sources of transformer power loss.

6. A transformer has N_p = 3600 turns, N_s = 600 turns, and R_s = 50 Ω. Calculate R_s.

7. If a transformer has N_p = 4000 turns, N_s = 2000 turns, and R_s = 50 Ω, what are the primary and secondary currents (I_p and I_s) if V = 100V? Assume 100% efficiency.

REFERENCES

Boylestad, Robert L. Introductory Circuit Analysis. 3rd ed. Columbus, OH: Charles E. Merrill Publishing Co., 1977.

Harris, Norman C. and Hemmerling, Edwin M. Introductory Applied Physics. 3rd ed. New York: McGraw-Hill, 1972.

Olivo, C. Thomas. Fundamentals of Applied Physics. Albany, NY: Delmar Publishers, Inc., 1977.

Zebrowski, Ernest, Jr. Physics for Technicians. New York: McGraw-Hill, 1974.